M O D U L A R S C A L E S

–A S Y N O P S I S–

Based on the Ancient Modes

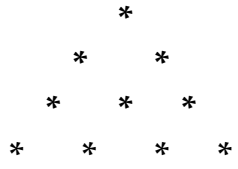
as

Derived from

The

P E R F E C T F O U R T H

Expressed in Quarter Tones



S.S. **F.T.S.**

2017 - 2022

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Perfect Fourth expressed in quarter-tones.

Ancient Modal Systems.

Ancient Perfect System.

Scales based on the Ancient Modal System.

Platonic considerations.

The natural interval series.

Numbers - Integers

Master Tunings for "flexible tonics".

Numbers, Integers and Frequencies for electronic music .

The relationship of the baroque "church" modes to *one* of the ancient modal systems.

Scalar bases for harmonic exploration.

Frequency charts.

THE PERFECT FOURTH

Expressed in

QUARTER-TONES

It is commonly known that the interval of the perfect fourth – the 1st consonant available as "discovered by Pythagoras"- , was reached using three genus methods, or system types – modes (mod-ulation) – not to be confused with the baroque so-called "Church" (Greek-Tribal-named) modes.
The basic tetrachord theory according to Theon of Smyrna and Aristoxenes states that the intervals obtained, were based upon the natural modulation of a voice going from an initial first sound/note/tone, to a higher tone, producing the interval of a half-tone; then going again higher through the interval of a tone; and when continuing to modulate cannot attain any other interval than that of another tone.
This higher consonant tone, together with the initial tone, gives the consonant of the perfect fourth [–*inverted major fifth interval]*.
This modulation method is called the "tetrachord system", the particular system consisting of three intervals of a half-tone, a tone and another tone is named the "Diatonic" system, which is also applied to the "Harmony of the Spheres".
Two other systems of modulation to the fourth are also noted:
The "Chromatic"; consists of rising to a half-tone, a half-tone again and then higher through three half-tones to reach the fourth.
The "Enharmonic"; rises a half-half-tone (i.e. one quarter-tone), another one quarter-tone and finally a double-tone (two tones).
These commonly known interval modulations are represented below.

Diatonic	1/2 tone	1 tone	1 tone
Chromatic	1/2 tone	1/2 tone	1&1/2 tones, i.e. 3 half-tones
Enharmonic	1/4 tone	1/4 tone	2 tones

Plato regarded the Diatonic mode as the most noble and more natural.
Aristoxenes regarded the Enharmonic mode as the most difficult requiring much art and study, his pupils regarded the quarter tone as the smallest appreciable interval.
Theon also, states that all consonances are contained within the quaternary [1 2 3 4] , the fourth, fifth, the octave, octave and fifth and the double octave, the ratios, 4:3, 3:2 and the double triple quadruple ratios.

The smallest interval noted above is the quarter-tone and the number of quarter-tones required to make up the interval of a fourth is **10**; – The "Pythagorean mystic *Decade"* i.e. 1+2+3+4 = 10 or IO… a matter for further investigation for any interested in truly liberating thoughts, studies and pursuits.

The table above can be be expressed as below in quantities of quarter-tones.

Diatonic	2	4	4
Chromatic	2	2	6
Enharmonic	1	1	8

Simplified numerically as. . .

2 4 4
2 2 6
1 1 8

…and recognise that the sequences, –expressed in quarter tones– for the systems, or ancient modes, are two repetitions of one number, and the third number is that required to make up the total to ten. These numbers are permutations of a numerical sequence of terms totalling 10. (the perfect fourth consisting of ten quarter-tone intervals) the numbers above can be reorganised in numerical sequences…

2 4 4 244 permutation of 442 4 4 2
2 2 6 reorganised as… 2 2 6 and again sorted numerically… 1 1 8 2 2 6 4 4 2
1 1 8 1 1 8

The inclusion of another set of quarter-tone intervals "***3 3 4***" i.e. one half-tone and a quarter-tone i.e. 3 quarter-tones, repeating another 3 quarter-tones and finally adding one whole tone i.e. 4 quarter-tones, to make up the interval of a fourth producing another tetrachord that completes the Pythagorean 1 2 3 4 sequence appears logical and not unreasonable.

1 1 8
2 2 6
3 3 4
4 4 2

Theon´s statement …"that all consonances are contained within the quaternary 1 2 3 4 etc. … is another reason not to exclude the 3 3 4 mode from thus completing the set of intervals as expressed in quarter-tones required to produce the 1st consonant of the fourth, as the complete set itself is based upon and displays the quaternary.

As the quarter-tone itself is included in the original "Enharmonic" ancient system or mode, and that the quarter-tone is almost universally accepted, frequently used by "serious" modern composers, and, its inclusion within the (electronic) frequency domain being relatively simple, this "extra" unnamed 3 3 4 mode will now be included and taken into account.

That the quarter tone is common in Byzantine music (which uses modes 118, 226 & 442), as it is in Arab and Persian music is common knowledge, not forgetting other non-"European" music where the quarter tone traditionally also occurs.

All four modes with their permutations can now be represented in terms of quarter tones.

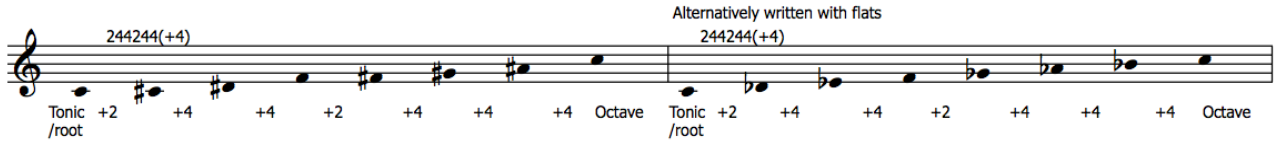
1 1 8	2 2 6	3 3 4	4 4 2
1 8 1	2 6 2	3 4 3	4 2 4
8 1 1	6 2 2	4 3 3	2 4 4
A	B	Γ	Δ

The table above shows the 12 possible sets of intervals for scale bases or with which intervals to "tune up or down" to using the 24 note octave with a potentially infinite number of ***frequencies*** as "tonic/root note".
To obtain the octave (as 8th note) for a one octave scale, the chosen interval set was repeated and an extra 1 tone interval (4 quarter-tones) was then added to the 7th note.
Modes A & Γ contain ¼ tones, modes B & Δ the 12 commonly known tempered notes only.

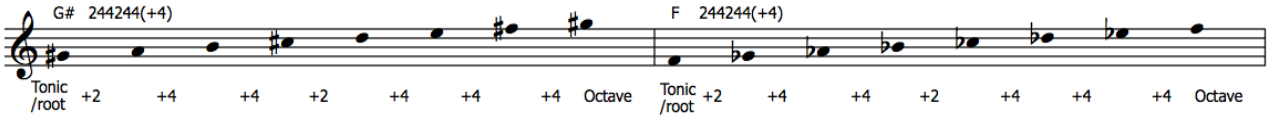
Tuning individual notes (or strings) is undertaken by using the interval sets. Here for example, expressed in ¼ tones, from the first table – "Diatonic"- half tone, tone, tone, repeated, plus an extra tone to reach the octave.

In quarter-tones:
2 4 4 2 4 4 + 4 (permutation of 442)

1st Note	Base tone	("Tonic/Root")
2nd Note	Half tone	2 quarter tones
3rd Note	Tone	4 " "
4th Note	Tone	4 " "
5th Note	Half tone	2 " "
6th Note	Tone	4 " "
7th Note	Tone	4 " "
8th Note	Tone	4 " " - Octave (8ve) of 1st Note.



"Diatonic"on other root notes:



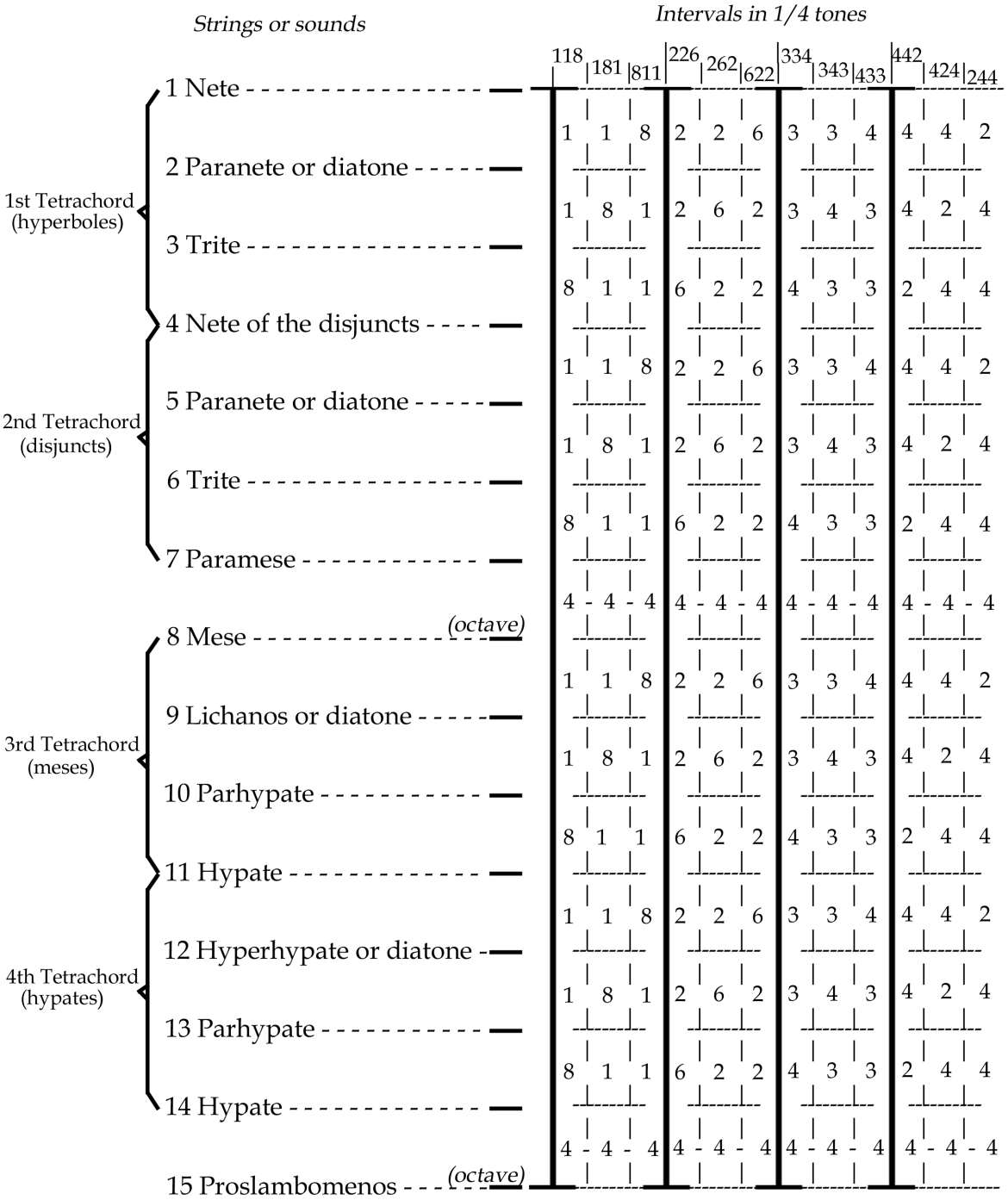
This same modal system with slight changes, occurs within the baroque "church modes" as will be shown further.

The ancient "Perfect System" was made up of two octaves as an extension of the human voice, formed of four small systems each consisting of four frequencies/notes/tones. the extremities of which gave the governing consonant of the fourth. The small systems were called tetrachords because the initial tuning of the notes were given by (according-to-mode-pre-tuned) four-stringed lyres or harps.

Below is a representation of the ancient "Perfect system" the intervals (divisions of the strings) are disposed vertically.

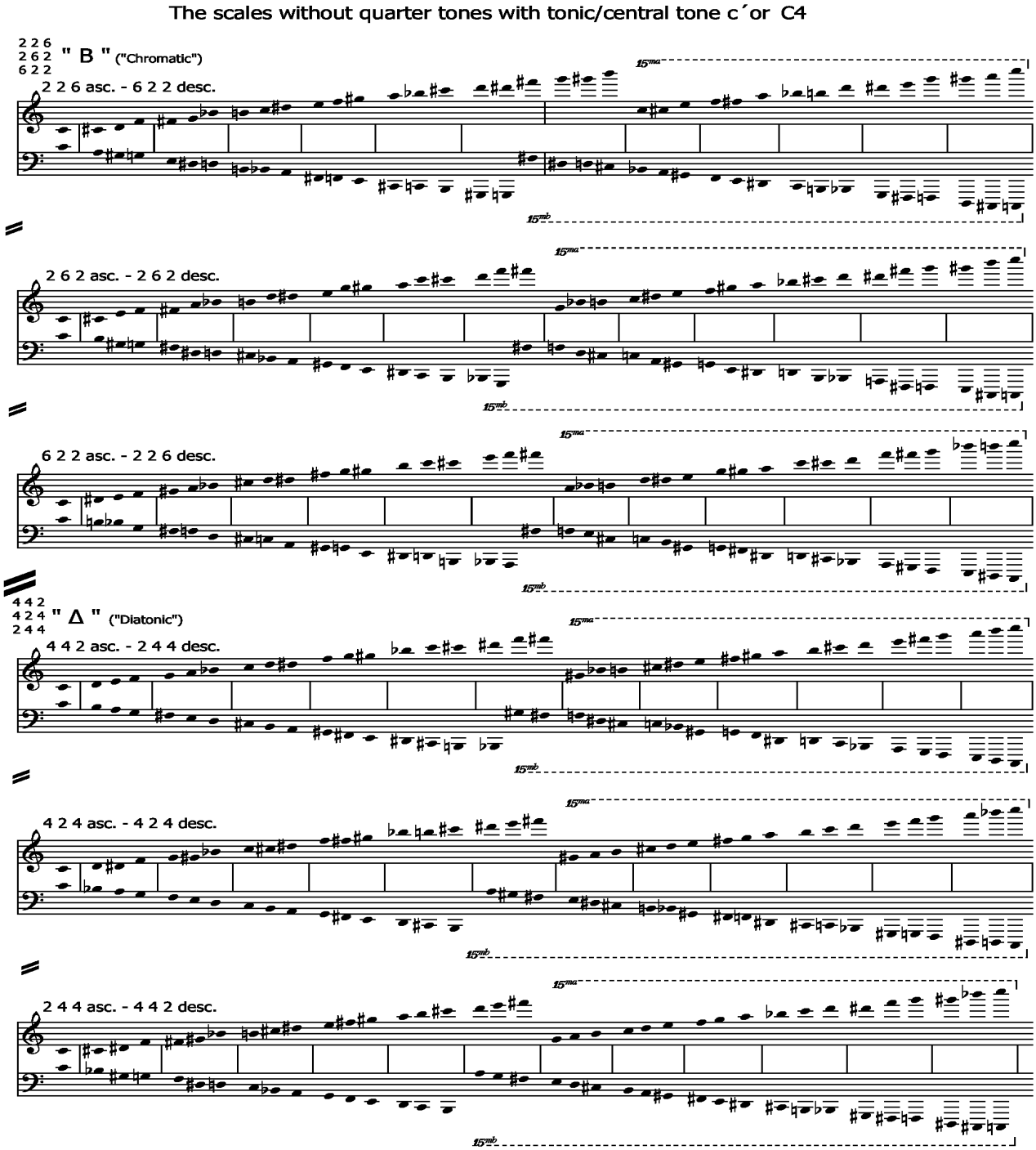
It is not here intended to go into technical details regarding this particular system (string names etc.), as this work is not about the ancient system as such; this diagram – including the " Γ " 334 intervals, is to display that twelve two octave scales were/are available within the ancient setup.

"Perfect system formed of two octaves containing all four modes/species (& permutations)


Strings or sounds
Intervals in 1/4 tones
118 181 811 226 262 622 334 343 433 442 424 244
1 Nete
2 Paranete or diatone
3 Trite
4 Nete of the disjuncts
5 Paranete or diatone
6 Trite
7 Paramese
8 Mese
9 Lichanos or diatone
10 Parhypate
11 Hypate
12 Hyperhypate or diatone
13 Parhypate
14 Hypate
15 Proslambomenos
(octave)
(octave)
1st Tetrachord (hyperboles)
2nd Tetrachord (disjuncts)
3rd Tetrachord (meses)
4th Tetrachord (hypates)
1 1 8 2 2 6 3 3 4 4 4 2
1 8 1 2 6 2 3 4 3 4 2 4
8 1 1 6 2 2 4 3 3 2 4 4
4 - 4 - 4 4 - 4 - 4 4 - 4 - 4 4 - 4 - 4

The scales are descending.
No. 15 Proslabomenos is an added extra string for the lowest note.

It is also possible to repeat the modal intervals without the inclusion of the required extra tone for a one octave scale, creating a continuous scale for one particular mode; applicable to all the ancient modes (118 etc.) by using a "central note/tone" of the tempered notes around middle c´/C4, i.e from f/F3 to e´/E4.


The scales without quarter tones with tonic/central tone c´or C4
2 2 6
2 6 2
6 2 2
" B " ("Chromatic")
2 2 6 asc. - 6 2 2 desc.
2 6 2 asc. - 2 6 2 desc.
6 2 2 asc. - 2 2 6 desc.
4 4 2
4 2 4
2 4 4
" Δ " ("Diatonic")
4 4 2 asc. - 2 4 4 desc.
4 2 4 asc. - 4 2 4 desc.
2 4 4 asc. - 4 4 2 desc.

The unique modal sequences encompass a range of five octaves; from the central tone c´ upwards to c´´´´´´ and another five octaves up from the low CCCC to c´; essentially a repetition. From the low CCCC up to central c´ there is a five octave compass and again from central c´ascending to c´´´´´´ the note orders are repeated (as from CCCC upwards), thereby encompassing ten octaves in total. N.B. when reading from c´ down to CCCC or from c´´´´´´ down to c´ the interval sequence is reversed e.g. 244 ascending is 442 descending, –except modes 262 & 424 (181 & 343) where the sequence order is identical–.

The scales types below on central root note c´ or MIDI C4 are marked as ascending; e.g. 118 - "Enharmonic" - ascends continuously by intervals of 118, from the lowest note, but also continuously descends by intervals of 811 from the highest note.
All 12 notes of the tempered scale are present and a selection of quarter-tones are available in modes 118 & 334 separately. The B flats, (as are any other enharmonic flats) in these scales are "resolved" to A sharp raisings (and any other applicable enharmonic raisings) in order to minimize the quarter tone accidentals to natural ¼-tone raised and sharp ¼ -tone raised only; raised notes exclusively are notated throughout.

All 24 notes within the ¼ tone octave are present in the *complete* set of four scales/modes. The sequences again extend ten octaves in total. From a low CCCC upwards five octaves to c´, this is the first completed sequence from the lowest note. Also, c´ has a central position as notation on staves where the sequence is repeated upwards another five octaves where it completes its cycle at c´´´´´´ .

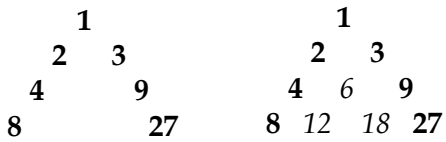
A - 118 asc. = 811 desc.
B - 226 asc. = 622 desc.
Γ - 334 asc. = 433 desc.
Δ - 442 asc. = 244 desc.

As for the reckoning of the actual frequency values or "tonics" upon which to tune the intervals from, the following from Plato is not amiss...

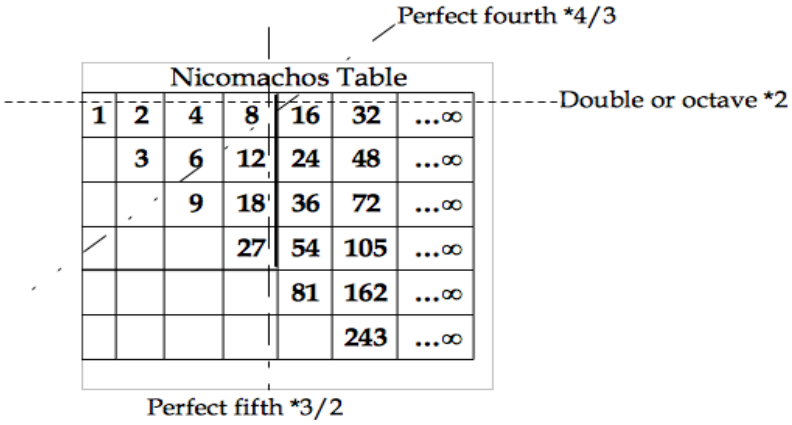
... "Now God [Demiourgos] did not make the soul after the body,
although we are speaking of them in this order; for having brought them together
he would never have allowed that the elder should be ruled by the younger;
but this is a random manner of speaking which we have,
because somehow we ourselves too are very much under the dominion of chance.
Whereas he made the soul in origin and excellence prior to and older than the body,
to be the ruler and mistress, of whom the body was to be the subject.
And he made her out of the following elements and on this wise:
Out of the indivisible and unchangeable,
and also out of that which is divisible and has to do with material bodies,
he compounded a third and intermediate kind of essence,
partaking of the nature of the same and of the other,
and this compound he placed accordingly in a mean
between the indivisible, and the divisible and material.
He took the three elements of the same, the other, and the essence, and mingled them into one form,
compressing by force the reluctant and unsociable nature of the other into the same.
When he had mingled them with the essence and out of three made one,
he again divided this whole into as many portions as was fitting,
each portion being a compound of the same, the other, and the essence.
And he proceeded to divide after this manner:-
First of all, he took away one part of the whole [1],
and then he separated a second part which was double the first [2],
and then he took away a third part
which was half as much again as the second and three times as much as the first [3],
and then he took a fourth part which was twice as much as the second [4],
and a fifth part which was three times the third [9], and a sixth part which was eight times the first [8],
and a seventh part which was twenty-seven times the first [27].
After this he filled up the double intervals [i.e. between 1, 2, 4, 8]
and the triple [i.e. between 1, 3, 9, 27] cutting off yet other portions from the mixture
and placing them in the intervals, so that in each interval there were two kinds of means,
the one exceeding and exceeded by equal parts of its extremes
[as for example 1, 4/3, 2, in which the mean 4/3 is one-third of 1 more than 1, and one-third of 2 less than2],
the other being that kind of mean which exceeds and is exceeded by an equal number.
Where there were intervals of 3/2 and of 4/3 and of 9/8,
made by the connecting terms in the former intervals,
he filled up all the intervals of 4/3 with the interval of 9/8,
leaving a fraction over; and the interval which this fraction expressed was in the ratio of 256 to 243.
And thus the whole mixture out of which he cut these portions was all exhausted by him.
This entire compound he divided lengthways into two parts,
which he joined to one another at the centre like the letter X,
and bent them into a circular form, connecting them with themselves
and each other at the point opposite to their original meeting-point;
and, comprehending them in a uniform revolution upon the same axis,
he made the one the outer and the other the inner circle.
Now the motion of the outer circle he called the motion of the same,
and the motion of the inner circle the motion of the other or diverse.
The motion of the same he carried round by the side to the right,
and the motion of the diverse diagonally to the left.
And he gave dominion to the motion of the same and like,
for that he left single and undivided;
but the inner motion he divided in six places and made seven unequal circles
having their intervals in ratios of two-and three, three of each,
and bade the orbits proceed in a direction opposite to one another;
and three [Sun, Mercury, Venus] he made to move with equal swiftness,
and the remaining four [Moon, Saturn, Mars, Jupiter]
to move with unequal swiftness to the three and to one another,
but in due proportion." ...

"Timaeus": Rev. Jowett´s translation

The above, has amongst many other things also led to the so-called "Lambda (**Λ**) of Plato" which contains the Arithmetic Means - the central nos. *6,12,18* in *italics;* the Geometric Means - the nos. 2,4,8 on the left; and the Harmonic Means - the nos. 3,9,27 on the right.



Calculating by simple arithmetic the doubling relationship progressions the following "Nicomachos Table" is derived, which Nicomachos himself considered a "theorem replete with intellectual conception" useful for the purpose of understanding the Platonic generation of the soul in the Timaeus, and the intervals of the harmonic discipline.



From the above "Nichomachos Table":
Horizontally adjacent numbers progress in double or octave relationships.
Diagonally adjacent numbers display the perfect fourth relationship (4/3).
Vertically adjacent numbers display the perfect fifth relationship (3/2).
Taking three adjacent horizontal numbers e.g. 1 2 4 the Geometric Mean [√ab] is the number in the middle i.e. 2, or from the 2nd horizontal row GM of 3 & 12 = 6 .
The Arithmetic Mean [a+b/2] of two adjacent horizontal numbers is displayed by the number directly beneath the first term, e.g. AM of 4 & 8 = 6; AM of 6 & 12 = 9 .
The Harmonic Mean [2ab/a+b] of two adjacent horizontal numbers is displayed by the number directly above the second term, e.g. HM of 6 & 12 = 8, HM of 9 & 18 = 12 .
J. Dupuis the French translator of Theon´s work in 1892, from the preceding considerations, gives in his notes a proportionally progressing table similar to the one below.

	1	**2**	**4**	**8**	**16**
	9/8	9/4	9/2	9	18
	81/64	81/32	81/16	81/8	81/4
Harmonic means---	4/3	8/3	16/3	32/3	64/3
Arithmetic means--	3/2	**3**	**6**	**12**	**24**
	27/16	27/8	27/4	27/2	**27**...*(Plato´s Lamda -Λ- Ceases here).*
	243/128	243/64	243/32	243/16	243/8
	2	**4**	**8**	**16**	**32**

The above is a representation of the first five octaves divided into 7 steps - "**Pythagorean Intervals**".

For the first octave the multiple is 1 for the next octave 2, and so on, the large table below lists the numbers for 21 octaves.

1	1	X 1 = 1
9:8	9 : 8	X 1 = 1.125
81:64	81 : 64	X 1 = 1.26527
4:3 – Harmonic means	4 : 3 – Harmonic means	X 1 = 1.333333
3:2 – Arithmetic means	3 : 2 – Arithmetic means	X 1 = 1.5
27:16	27 : 16	X 1 = 1.6875
243:128	243 : 128	X 1 = 1.8984375
2	2 : 1	X 1 = 2

Also named "The Natural Interval Series".

The results in this table are calculated to the 22nd octave, from 1.

Some if not many of these "natural numbers" will not be unfamiliar to those working with electronic music, music programmes, synthesizers and even computer programming.

This list of number values in themselves should be of interest as they can also be taken as a list of frequencies, which in a sense is what they are. When in the audible frequency range, a "scale" or Thromos ΔΡΟΜΟΣ (road-way) of seven tones/notes in "Pythagorean Intervals" can be constructed from these values. From this table one notes that a´- MIDI A4 can or should be at a frequency of 423Hz instead of the standard 440Hz, which seems rather arbitrarily inept. Harmony or discord depends precisely upon the synchronous vibrations and complementary periods and considering that the other numbers and values relating to the ratios in musical harmony and for the calculation of the series itself are present in this list taking a value from here upon which to base initial tuning is neither counterproductive or counterintuitive.

Perhaps a middle c´ - MIDI C4 tuning of 256 Hz (instead of 261.626Hz) as a tuning basis.

Natural Interval Series

1	**54**	**2730.666667**	**147456**
1.125	**60.75**	**3072**	**165888**
1.265627	**64***	**3456**	**174762.6667**
1.333333	**72**	**3888**	**196608**
1.5	**81**	**4096***	**221184**
1.6875	**85.33333333**	**4608**	**248832**
1.8984375	**96**	**5184**	**262144***
2*	**108**	**5461.333333**	**294912**
2.25	**121.5**	**6144**	**331776**
2.53125	**128***	**6912**	**349525.3334**
2.666666667	**144**	**7776**	**393216**
3	**162**	**8192***	**442368**
3.375	**170.6666667**	**9216**	**497664**
3.796875	**192**	**10368**	**524288***
4*	**216**	**10922**	**589824.0001**
4.5	**243**	**12288**	**663552.0001**
5.0625	**256***	**13824**	**699050.6667**
5.333333333	**288**	**15552**	**786432.0001**
6	**324**	**16384***	**884736.0001**
6.75	**341.3333333**	**18432**	**995328.0001**
7.59375	**384**	**20736**	**1048576***
8*	**432**	**21845.33333**	**1179648**
9	**486**	**24576**	**1327104**
10.125	**512***	**27648**	**1398101.333**
10.6666667	**576**	**31104**	**1572864**
12	**648**	**32768***	**1769472**
13.5	**682.6666667**	**36864**	**1990656**
15.187	**768**	**41472**	**2097152***
16*	**864**	**43690.66667**	**2359296**
18	**972**	**49152**	**2654208**
20.25	**1024***	**55296**	**2796202.667**
21.33333333	**1152**	**62208**	**3145728**
24	**1296**	**65536***	**3538944**
27	**1365.333333**	**73728**	**3981312**
30.375	**1536**	**82944**	**4194304***
32*	**1728**	**87381.33334**	
36	**1944**	**98304.00001**	
40.5	**2048***	**110592**	
42.6666667	**2304**	**124416**	
48	**2592**	**131072***	

* octaves of/from 1

This 256Hz could otherwise be more suited for tuning the note d´ - MIDI D4 to, if everything should sound "shriller" and the rest be tuned/adjusted accordingly to this within the standard tempered tuning system. 440Hz minus 31.76 cents results in 432Hz. Many hardware digital synthesizers have a master tune setting. Analogue synths and equipment have complete frequency control. Sampled works "master tuned " to 440Hz can be detuned by 32 cents if the decimal numbers are unavailable.

For comparison shown below is a basic standard MIDI note number to Frequency table; read horizontally are the octaves from MIDI C2 to C4—"middle c"— (C-c´) The first column has the note name and the MIDI note number the other columns show only the MIDI note numbers and the frequency columns are the frequencies.

MIDI Note		Frequency	MIDI Note	Frequency	MIDI Note	Frequency
C	36	65.4063913251	48	130.8127826503	60	261.6255653006
Db	37	69.2956577442	49	138.5913154884	61	277.1826309769
D	38	73.4161919794	50	146.8323839587	62	293.6647679174
Eb	39	77.7817459305	51	155.5634918610	63	311.1269837221
E	40	82.4068892282	52	164.8137784564	64	329.6275569129
F	41	87.3070578583	53	174.6141157165	65	349.2282314330
G b	42	92.4986056779	54	184.9972113558	66	369.9944227116
G	43	97.9988589954	55	195.9977179909	67	391.9954359817
Ab	44	103.8261743950	56	207.6523487900	68	415.3046975799
A	45	110.0000000000	57	220.0000000000	69	440.0000000000
Bb	46	116.5409403795	58	233.0818807590	70	466.1637615181
B	47	123.4708253140	59	246.9416506281	71	493.8833012561

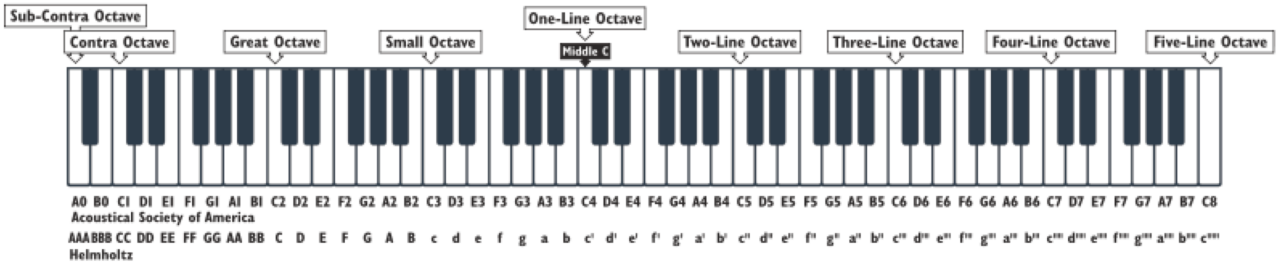
Converting Frequency to MIDI no.

$$m = 69 + 12 \cdot \log_2(f/440)$$

m = the required MIDI number:

f = the known frequency

substitute 432 for 440 in the above formula for harmonic MIDI frequencies(?)


Sub-Contra Octave
Contra Octave
Great Octave
Small Octave
One-Line Octave
Middle C
Two-Line Octave
Three-Line Octave
Four-Line Octave
Five-Line Octave
A0 B0 C1 D1 E1 F1 G1 A1 B1 C2 D2 E2 F2 G2 A2 B2 C3 D3 E3 F3 G3 A3 B3 C4 D4 E4 F4 G4 A4 B4 C5 D5 E5 F5 G5 A5 B5 C6 D6 E6 F6 G6 A6 B6 C7 D7 E7 F7 G7 A7 B7 C8
Acoustical Society of America
AAABBB CC DD EE FF GG AA BB C D E F G A B c d e f g a b c' d' e' f' g' a' b' c'' d'' e'' f'' g'' a'' b'' c'''
Helmholtz

For electronic works, the numbers/digits themselves as values can also represent ideal settings or starting points for whatever suitable purpose, within the relevant context to where the numbers can be applied within the programming parameters, e. g. frequencies themselves, frequency settings for eq., control rates for filters, envelopes, delays etc. For values just above zero as used in some software synthesis programmes, the numbers can be used with 0. before them or even 0.0, 0.00 etc. minus values can also be applied. A fairly simple coding example for SuperCollider is shown below.

Simple example: to illustrate timbre changes and other "subtleties" possible in electronic music by substituting commonly used nos., e.g. 440Hz, with Plato´s (or Pythagoras´s) "Natural Interval Series" numbers.
The following example is given here, without syntax colours or explanations about the code itself. **Super Collider** is used here as it is text based; the same principles can be applied to any software synthesis programme Pd, MaxMSP etc. i.e. for any others that can be directly controlled with numbers.
This example changes the spectrum but preserves the fundamental. With two simple number substitutions in the frequency domain resulting changes are clearly audible; to preserve the inherent relationships of the numbers to modal relationships it is important to discern where to use what values, and from which particular mode, when combining

[e.g. frequency] values from different tables or modes. The examples below can be copied directly into SuperCollider.
Example 1 is in its original setting, Example 2 has replacement values from the Natural Interval Series. After executing each example, audible differences are clearly heard.
N.B. no claim is made here for any intellectual property rights concerning the code as below.

```
//Ex. 1 - standard unaltered example using commonly used nos./values -------
(
~size = 4096; //common usage computer synthesis "standard no." also a Natural series octave value (see above in the Natural Interval Series).
~width = s.sampleRate / ~size;

SynthDef(\Timbre, { |freq = 440, amp = 0.03, binShift = 0|  //freq(uency) = 440Hz = a´- (not in the Natural Interval Series)
        var sig, pitch, chain;
        sig = Saw.ar(freq, amp);
        pitch = Pitch.kr(sig)[0];
        chain = FFT(LocalBuf(~size), sig);
        chain = PV_MagShift(chain, pitch / (pitch + ((binShift * ~width))), binShift);
        Out.ar(0, IFFT(chain) ! 2 * EnvGate())
}).add
)

x = Synth(\Timbre, [\freq, 600]); // freq(uency) = 600Hz = a commonly chosen round no.(not in the Natural Interval Series)
x.set(\binShift, 1);
x.set(\binShift, 2);
x.set(\binShift, 3);
x.set(\binShift, -2);
x.release
//
//Ex. 2 - same as above substituting 2 values --------

(
~size = 4096;
~width = s.sampleRate / ~size;

SynthDef(\Timbre, { |freq = 432, amp = 0.03, binShift = 0| //freq(ency) = 432Hz (from the Natural Interval Series; a near value to 440)
        var sig, pitch, chain;
        sig = Saw.ar(freq, amp);
        pitch = Pitch.kr(sig)[0];
        chain = FFT(LocalBuf(~size), sig);
        chain = PV_MagShift(chain, pitch / (pitch + ((binShift * ~width))), binShift);
        Out.ar(0, IFFT(chain) ! 2 * EnvGate())
}).add
)

x = Synth(\Timbre, [\freq, 642]); //freq(ency) = 642Hz (from the Natural Interval Series ; a near value to 600)
// below other values can be applied from the Natural Interval Series that are within the code relevant parameters.
x.set(\binShift, 1);// as in Ex. 1 above
x.set(\binShift, 2);//"          "
x.set(\binShift, 3);//"          "
x.set(\binShift, -2);// Minus value from above
x.set(\binShift, -1.125); //example  from Natural Interval Series as minus value
x.release
```

To return to the modes expressed as ¼ tone intervals within the notated tempered system.

A	B	Γ	Δ
1 1 8	2 2 6	3 3 4	4 4 2
1 8 1	2 6 2	3 4 3	4 2 4
8 1 1	6 2 2	4 3 3	2 4 4

As mentioned above with reference to the ancient "Perfect System", in order to tune instruments or separate strings, to achieve scales of one octave on whatever tonic or base frequency, the intervals of the modes and their permutations are arranged as shown below. The "mode" is repeated twice and then four quarter-tones are added to reach the octave.

Enharmonic A	Chromatic B	Γ	Diatonic Δ
118	226	334	442
1 1 8 1 1 8 - 4	2 2 6 2 2 6 - 4	3 3 4 3 3 4 - 4	4 4 2 4 4 2 - 4
1 8 1 1 8 1 - 4	2 6 2 2 6 2 - 4	3 4 3 3 4 3 - 4	4 2 4 4 2 4 - 4
8 1 1 8 1 1 - 4	6 2 2 6 2 2 - 4	4 3 3 4 3 3 - 4	2 4 4 2 4 4 - 4

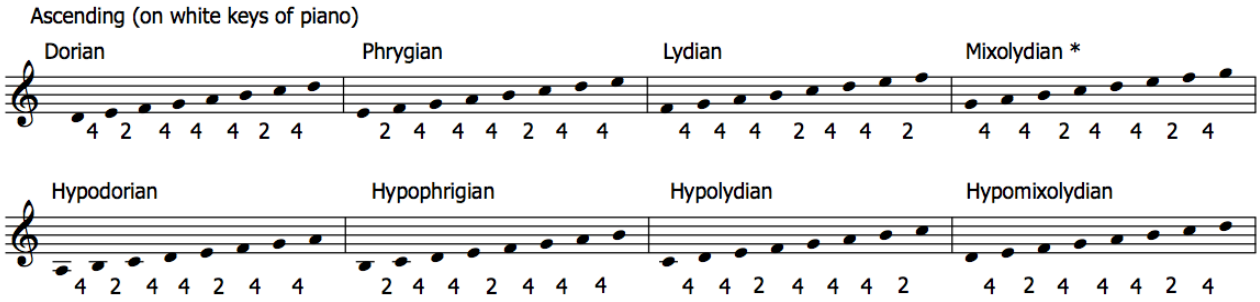
It is difficult to conceive that the unnamed "**Γ "** mode" (3 3 4) was unknown [arithmetically] to the ancients, as the enharmonic mode 1 1 8 was recognised and noted for its "difficulty to master"; bearing in mind that the "ancients" mention also their "ancients" who in turn no doubt had also their "ancients".
Also difficult to conceive is that ancient harpists were unaware of the various plucking techniques, the harmonics available on the individual strings of their instruments e.g. octave, fifth, double-octave, major-third, double-octave+fifth and minor-seventh (no doubt not named as such), and that melodic lines only were played.
And that music with simultaneously sounding chords (chord progressions) were never undertaken or heard of as has been suggested by some academics.

Below, the 4 modes are shown as 1 octave scales based on the note c´ with the intervals in ascending and descending order, the Diatonic or "**Δ"** mode exhibits a natural reversibility in the note orders of some of the scales, and the 433 mode has identical note ordering both ascending and descending.

Modes/Systems/Species ascending & descending scales - ΔPOMOI - (RoadWays)
Natural 1/4 tone higher
Sharp 1/4 tone higher
Natural 1/4 tone lower
Enharmonic " A "
1 1 8 1 1 8 + 4
1 8 1 1 8 1 + 4
8 1 1 8 1 1 + 4
Chromatic " B "
2 2 6 2 2 6 + 4
2 6 2 2 6 2 + 4
6 2 2 6 2 2 + 4
Unnamed " Γ "
3 3 4 3 3 4 + 4
3 4 3 3 4 3 + 4
4 3 3 4 3 3 + 4
Diatonic " Δ "
4 4 2 4 4 2 + 4
4 2 4 4 2 4 + 4
2 4 4 2 4 4 + 4
inherent reversability/reflection
* A# is resolved enharmonically to Bb
indicating the relationship
to classical harmony.

Within the Diatonic "**Δ**" system: the baroque "Church" or Greek-tribe-named modes are recognised by their interval usage.

Below is a typical text book or academic example of the baroque church modes with their assigned Greek-tribal-names. The intervals in ¼ tones are noted below between the notes.


Ascending (on white keys of piano)
Dorian
4 2 4 4 4 2 4
Phrygian
2 4 4 4 2 4 4
Lydian
4 4 4 2 4 4 2
Mixolydian *
4 4 2 4 4 2 4
Hypodorian
4 2 4 4 2 4 4
Hypophrigian
2 4 4 2 4 4 4
Hypolydian
4 4 2 4 4 4 2
Hypomixolydian
4 2 4 4 4 2 4

*Repetition of 1st interval set from "**Δ**" mode (4 4 2) **-** 4 4 2 4 4 2 (+ 4 to reach the octave).

The Hypomixlolydian mode has the same interval set as the Dorian.

The interval sets from above as a list (in quarter-tones). . .

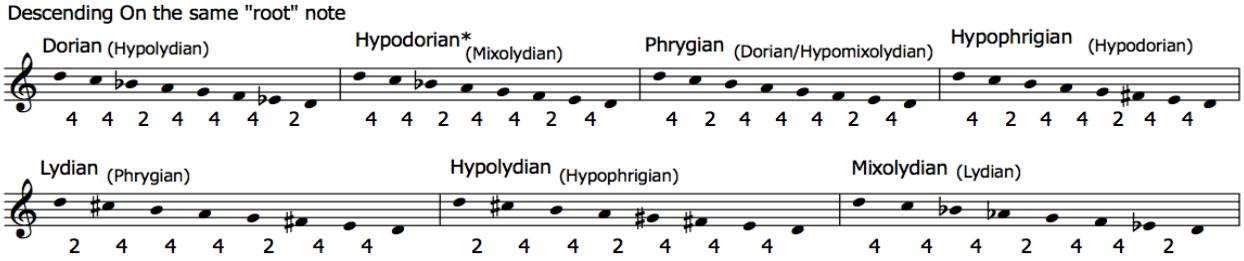
4 2 4 4 4 2 4 - Dorian
2 4 4 4 2 4 4 - Phrygian
4 4 4 2 4 4 2 - Lydian
4 4 2 4 4 2 4 - Mixolydian
4 2 4 4 2 4 4 - Hypodorian
2 4 4 2 4 4 4 - Hypophrygian
4 4 2 4 4 4 2 - Hypolidian

. . . are permutations of the above "Dorian" named interval set 4244424, which is an interval set permutation of the ancient "Δ " - Diatonic 442 mode.
The reordered permutations are listed below, with the original 4424424 as starting set.

4 4 2 4 4 2 4 - Mixolydian
4 2 4 4 2 4 4 - Hypodorian
2 4 4 2 4 4 4 - Hypophrygian
4 4 2 4 4 4 2 - Hypolidian
4 2 4 4 4 2 4 - Dorian
2 4 4 4 2 4 4 - Phrygian
4 4 4 2 4 4 2 - Lydian

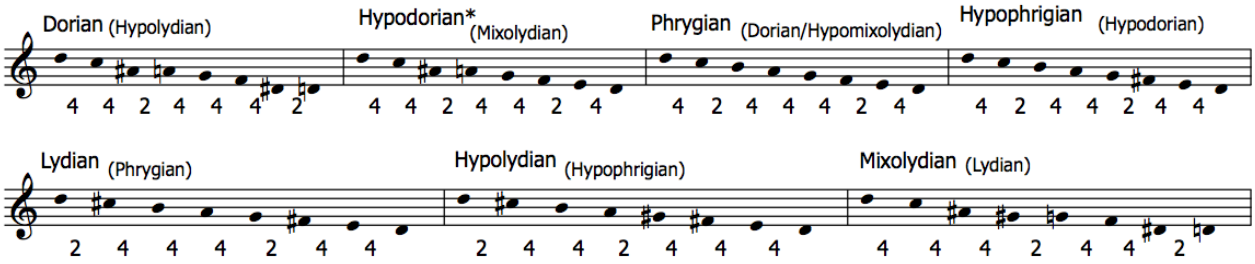
<u>Δ mode for 1 octave</u>
4 4 2 4 4 2 4
4 2 4 4 2 4 4
2 4 4 2 4 4 4

The original ancient modes/systems (as shown in the Modes/System/Species ascending and descending scales above), repeat and permutate while keeping intact the three initial intervals, after the repetition of the initial interval set only then is the extra interval of '4' - four 1/4 tones i.e. one whole tone, to reach the octave added.
The ancient scales were (supposedly) originally descending but the intervals as in the tables above also naturally produce ascending scales. Below the "church modes" descending to a tonic of d´ – D4.
There is no intention here to enter into any historical or traditional use and development regarding the baroque church modes; the focus is primarily on the intervallic usages and their permutations.


Descending On the same "root" note
Dorian (Hypolydian)
Hypodorian* (Mixolydian)
Phrygian (Dorian/Hypomixolydian)
Hypophrigian (Hypodorian)
4 4 2 4 4 4 2
4 4 2 4 4 2 4
4 2 4 4 4 2 4
4 2 4 4 2 4 4
Lydian (Phrygian)
Hypolydian (Hypophrigian)
Mixolydian (Lydian)
2 4 4 4 2 4 4
2 4 4 2 4 4 4
4 4 4 2 4 4 2

*Interval set 442 from "**Δ**" the modes in (brackets) are the corresponding ascending modes.

The use of "*enharmonic*" changes, by including flats is to show the classical music tempered scale/s and "harmonic" function derived from the "church modes" with the five lined staff system, all further notation will use only sharps and respective ¼ tone raisings, in the table below are the same descending set of modes with enharmonic changes showing sharps only.


Dorian (Hypolydian)
Hypodorian* (Mixolydian)
Phrygian (Dorian/Hypomixolydian)
Hypophrigian (Hypodorian)
4 4 2 4 4 4 2
4 4 2 4 4 2 4
4 2 4 4 4 2 4
4 2 4 4 2 4 4
Lydian (Phrygian)
Hypolydian (Hypophrigian)
Mixolydian (Lydian)
2 4 4 4 2 4 4
2 4 4 2 4 4 4
4 4 4 2 4 4 2

The corresponding ascending and descending interval set for the church/tribal modes is listed again below, where the cyclic nature of the baroque genus becomes clear and that the baroque church/greek-tribal-mode system is an artifice constructed by permutating the Diatonic interval set and including within the permutations themselves the extra 7th whole tone (four ¼ tones) interval required to reach the 8th note octave.
Next to the list below the original 4 4 2 "**Δ**" - Diatonic interval set is shown with their correspondingly named "church" modes, the extra interval '4' at the end of each interval set to reach the octave has been omitted.

442 asc. = ascending, desc. = descending

4 4 2 4 4 2 4 - Mixolydian asc. - Hypodorian desc.
4 2 4 4 2 4 4 - Hypodorian asc. - Hypophrigian desc.
2 4 4 2 4 4 4 - Hypophrigian asc. - Hypolydian desc.
4 4 2 4 4 4 2 - Hypolydian asc. - Dorian desc.
4 2 4 4 4 2 4 - Dorian asc. - Phrygian desc.
2 4 4 4 2 4 4 - Phrygian asc. - Lydian desc.
4 4 4 2 4 4 2 - Lydian asc. - Mixolydian desc.

Diatonic interval set
4 4 2
4 2 4
2 4 4
without extra interval (4) for Octave
---
4 4 2 4 4 2 - Mixolydian asc. - Dorian desc.
4 2 4 4 2 4 - Hypodorian asc - Hypophrigian desc.
2 4 4 2 4 4 - Hypophrigian asc. - Hypolydian desc.

When the permutations are as such for the Diatonic "**Δ**" intervals that produce the "academic" church modes, it is therefore also valid that the remaining three "**A**", "**B**", and "**Γ**" modal interval sets can be permutated in a similar manner as the above "church modes", producing thereby their own set of respective (baroque) ""church" Greek - tribe - name - modes"".

The complete “church mode” (permutated) set for all four modal interval sets.

118 A	226 B	334 Γ	442 Δ
1 1 8 1 1 8 4	2 2 6 2 2 6 4	3 3 4 3 3 4 4	4 4 2 4 4 2 4
1 8 1 1 8 4 1	2 6 2 2 6 4 2	3 4 3 3 4 4 3	4 2 4 4 2 4 4
8 1 1 8 4 1 1	6 2 2 6 4 2 2	4 3 3 4 4 3 3	2 4 4 2 4 4 4
1 1 8 4 1 1 8	2 2 6 4 2 2 6	3 3 4 4 3 3 4	4 4 2 4 4 4 2
1 8 4 1 1 8 1	2 6 4 2 2 6 2	3 4 4 3 3 4 3	4 2 4 4 4 2 4
8 4 1 1 8 1 1	6 4 2 2 6 2 2	4 4 3 3 4 3 3	2 4 4 4 2 4 4
4 1 1 8 1 1 8	4 2 2 6 2 2 6	4 3 3 4 3 3 4	4 4 4 2 4 4 2

♮ natural 1/4 tone higher

♯ sharp 1/4 tone higher

"CHURCH" or "TRIBAL" MODES on d´ (ascending & descending)

Enharmonic " A " octave scales - 1181184 etc.

" A "

Chromatic " B " octave scales - 2262264 etc.

" B "

" Γ " octave scales - 3343344 etc.

" Γ "

Diatonic " Δ " octave scales - 4424424 etc.

" Δ "

Natural 1/4 tone higher

Sharp 1/4 tone higher

"CHURCH" or "TRIBAL" MODES on c´ ascending & descending

Enharmonic " A " octave scales - 1181184 permutations.

" A "

Chromatic " B " octave scales - 2262264 permutations.

" B "

" Γ " octave scales - 3343344 permutations.

" Γ "

Diatonic " Δ " octave scales - 4424424 permutations.

" Δ "

FREQUENCIES

The Harmonic Series, expressed mathematically as the divergent infinite series...

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots.$$

The formula above shows the Harmonic Series expressed mathematically as the divergent series. "The series acquired from adding the same Hertz value repeatedly, which results in a string of intervals of decreasing size, is called the Harmonic Series."
To obtain the ¼ tone frequency list for the following tables for frequencies from 1Hz - 1048576 Hz the formula "**expr $f2*(pow (2, ($f1/24)))** " was used within pd –pure data– sound synthesis programme, to calculate the intervals for equal-tempered intervals within a 24 tone/step octave. The fundamental frequency, the 2nd variable ($f), was set to **1**(Hz) instead of the usual "tempered" 440(Hz) to enable calculation from 1(Hz - cycle per second) as ***the*** fundamental frequency as in the Natural Interval Series above as this seemed most rationally natural. The frequencies in the tables below were calculated from this initial frequency of 1 (unity).

1/4 tone Frequency List - 24 terms per octave- 20 octaves.

When read horizontally the frequencies are doubled.

1	**2**	4	8	**16**	**32**	**64**	**128**	**256**	**512**	**1024**
1.0293	**2.0586**	**4.11721**	**8.23442**	**16.4688**	**32.9377**	**65.8753**	**131.751**	**263.501**	**527.003**	**1054.01**
1.05946	**2.11893**	**4.23785**	**8.4757**	**16.9514**	**33.9028**	**67.8056**	**135.611**	**271.223**	**542.445**	**1084.89**
1.09051	**2.18102**	**4.36203**	**8.72406**	**17.4481**	**34.8962**	**69.7925**	**139.585**	**279.17**	**558.34**	**1116.68**
1.12246	**2.24492**	**4.48985**	**8.9797**	**17.9594**	**35.9188**	**71.8376**	**143.675**	**287.35**	**574.701**	**1149.4**
1.15535	**2.31071**	**4.62141**	**9.24282**	**18.4856**	**36.9713**	**73.9426**	**147.885**	**295.77**	**591.54**	**1183.08**
1.18921	**2.37841**	**4.75683**	**9.51366**	**19.0273**	**38.0546**	**76.1093**	**152.219**	**304.437**	**608.874**	**1217.75**
1.22405	**2.44811**	**4.89621**	**9.79243**	**19.5849**	**39.1697**	**78.3394**	**156.679**	**313.358**	**626.716**	**1253.43**
1.25992	**2.51984**	**5.03968**	**10.0794**	**20.1587**	**40.3175**	**80.635**	**161.27**	**322.54**	**645.079**	**1290.16**
1.29684	**2.59368**	**5.18736**	**10.3747**	**20.7494**	**41.4989**	**82.9977**	**165.995**	**331.991**	**663.982**	**1327.96**
1.33484	**2.66968**	**5.33936**	**10.6787**	**21.3574**	**42.7149**	**85.4297**	**170.859**	**341.719**	**683.438**	**1366.88**
1.37395	**2.74791**	**5.49581**	**10.9916**	**21.9833**	**43.9665**	**87.933**	**175.866**	**351.732**	**703.464**	**1406.93**
1.41421	**2.82843**	**5.65685**	**11.3137**	**22.6274**	**45.2548**	**90.5097**	**181.019**	**362.039**	**724.077**	**1448.15**
1.45565	**2.91131**	**5.82261**	**11.6452**	**23.2904**	**46.5809**	**93.1618**	**186.324**	**372.647**	**745.295**	**1490.59**
1.49831	**2.99661**	**5.99323**	**11.9865**	**23.9729**	**47.9458**	**95.8917**	**191.783**	**383.567**	**767.133**	**1534.27**
1.54221	**3.08442**	**6.16884**	**12.3377**	**24.6754**	**49.3507**	**98.7015**	**197.403**	**394.806**	**789.612**	**1579.22**
1.5874	**3.1748**	**6.3496**	**12.6992**	**25.3984**	**50.7968**	**101.594**	**203.187**	**406.375**	**812.75**	**1625.5**
1.63392	**3.26783**	**6.53566**	**13.0713**	**26.1427**	**52.2853**	**104.571**	**209.141**	**418.282**	**836.565**	**1673.13**
1.68179	**3.36359**	**6.72717**	**13.4543**	**26.9087**	**53.8174**	**107.635**	**215.269**	**430.539**	**861.078**	**1722.16**
1.73107	**3.46215**	**6.92429**	**13.8486**	**27.6972**	**55.3943**	**110.789**	**221.577**	**443.155**	**886.31**	**1772.62**
1.7818	**3.5636**	**7.12719**	**14.2544**	**28.5088**	**57.0175**	**114.035**	**228.07**	**456.14**	**912.28**	**1824.56**
1.83401	**3.66802**	**7.33603**	**14.6721**	**29.3441**	**58.6883**	**117.377**	**234.753**	**469.506**	**939.012**	**1878.02**
1.88775	**3.7755**	**7.55099**	**15.102**	**30.204**	**60.4079**	**120.816**	**241.632**	**483.264**	**966.528**	**1933.06**
1.94306	**3.88613**	**7.77225**	**15.5445**	**31.089**	**62.1781**	**124.356**	**248.712**	**497.424**	**994.849**	**1989.7**
2	**4**	**8**	**16**	**32**	**64**	**128**	**256**	**512**	**1024**	**2048**

List continues below; from 2048

continuation from above:

2048	4096	8192	16384	32768	65536	131072	262144	524288
2108.01	4216.02	8432.05	16864.1	33728.2	67456.3	134913	269825	539651
2169.78	4339.56	8679.12	17358.2	34716.5	69433	138866	277732	555464
2233.36	4466.72	8933.44	17866.9	35733.8	71467.5	142935	285870	571740
2298.8	4597.61	9195.21	18390.4	36780.8	73561.6	147123	294247	588493
2366.16	4732.32	9464.65	18929.3	37858.6	75717.2	151434	302869	605738
2435.5	4870.99	9741.98	19484	38967.9	77935.9	155872	311744	623487
2506.86	5013.72	10027.4	20054.9	40109.8	80219.5	160439	320878	641756
2580.32	5160.64	10321.3	20642.5	41285.1	82570.2	165140	330281	660562
2655.93	5311.85	10623.7	21247.4	42494.8	84989.7	169979	339959	679917
2733.75	5467.51	10935	21870	43740	87480	174960	349920	699840
2813.86	5627.71	11255.4	22510.9	45021.7	90043.5	180087	360174	720348
2896.31	5792.62	11585.2	23170.5	46340.9	92681.9	185364	370728	741455
2981.18	5962.36	11924.7	23849.4	47698.9	95397.6	190795	381591	763181
3068.53	6137.06	12274.1	24548.3	49096.5	98193.1	196386	392772	785545
3158.45	6316.9	12633.8	25267.6	50535.2	101070	202141	404281	808563
3251	6502	13004	26008	52016	104032	208064	416127	832255
3346.26	6692.52	13385	26770.1	53540.1	107080	214161	428321	856643
3444.31	6888.62	13777.2	27554.5	55109	110218	220436	440872	881744
3545.24	7090.48	14181	28361.9	56723.8	113448	226895	453790	907580
3649.12	7298.24	14596.5	29193	58385.9	116772	233544	467088	934175
3756.05	7512.1	15024.2	30048.4	60096.8	120194	240387	480774	961548
3866.11	7732.22	15464.4	30928.9	61857.8	123715	247431	494862	989724
3979.39	7958.79	15917.6	31835.2	63670.3	127341	254681	509363	1018730
4096	8192	16384	32768	65536	131072	262144	524288	1048576

The tables below show the resulting frequencies for the four initial "ancient modes" as Hertz values.
The frequencies for these are repeated continuously e.g. 118118118 . . . ∞ in equal-tempered steps, with their permutations, from 1 - 32768 Hz (109 frequencies).
Every third frequency (number) after the initial (1st) value in the column is the interval of a fourth and therefore common to all modes and their permutations. Many other frequencies are also common across the modes.

Ancient Modes

A	B	Γ	Δ
1 1 8	2 2 6	3 3 4	4 4 2
1 8 1	2 6 2	3 4 3	4 2 4
8 1 1	6 2 2	4 3 3	2 4 4

ENHARMONIC "A" 118, 181, 811.

118

1	**3.1748**	**10.0794**	**32**	**101.594**	**322.54**	**1024**	**3251**	**10321.3**
1.0293	**3.26783**	**10.3747**	**32.9377**	**104.571**	**331.991**	**1054.01**	**3346.26**	**10623.7**
1.05946	**3.36359**	**10.6787**	**33.9028**	**107.635**	**341.719**	**1084.89**	**3444.31**	**10935**
1.33484	**4.23785**	**13.4543**	**42.7149**	**135.611**	**430.539**	**1366.88**	**4339.56**	**13777.2**
1.37395	**4.36203**	**13.8486**	**43.9665**	**139.585**	**443.155**	**1406.93**	**4466.72**	**14181**
1.41421	**4.48985**	**14.2544**	**45.2548**	**143.675**	**456.14**	**1448.15**	**4597.61**	**14596.5**
1.7818	**5.65685**	**17.9594**	**57.0175**	**181.019**	**574.701**	**1824.56**	**5792.62**	**18390.4**
1.83401	**5.82261**	**18.4856**	**58.6883**	**186.324**	**591.54**	**1878.02**	**5962.36**	**18929.3**
1.88775	**5.99323**	**19.0273**	**60.4079**	**191.783**	**608.874**	**1933.06**	**6137.06**	**19484**
2.37841	**7.55099**	**23.9729**	**76.1093**	**241.632**	**767.133**	**2435.5**	**7732.22**	**24548.3**
2.44811	**7.77225**	**24.6754**	**78.3394**	**248.712**	**789.612**	**2506.86**	**7958.79**	**25267.6**
2.51984	**8**	**25.3984**	**80.635**	**256**	**812.75**	**2580.32**	**8192**	**26008**
								32768

181

1	3.1748	10.0794	32	101.594	322.54	1024	3251	10321.3
1.0293	3.26783	10.3747	32.9377	104.571	331.991	1054.01	3346.26	10623.7
1.29684	4.11721	13.0713	41.4989	131.751	418.282	1327.96	4216.02	13385
1.33484	4.23785	13.4543	42.7149	135.611	430.539	1366.88	4339.56	13777.2
1.37395	4.36203	13.8486	43.9665	139.585	443.155	1406.93	4466.72	14181
1.73107	5.49581	17.4481	55.3943	175.866	558.34	1772.62	5627.71	17866.9
1.7818	5.65685	17.9594	57.0175	181.019	574.701	1824.56	5792.62	18390.4
1.83401	5.82261	18.4856	58.6883	186.324	591.54	1878.02	5962.36	18929.3
2.31071	7.33603	23.2904	73.9426	234.753	745.295	2366.16	7512.1	23849.4
2.37841	7.55099	23.9729	76.1093	241.632	767.133	2435.5	7732.22	24548.3
2.44811	7.77225	24.6754	78.3394	248.712	789.612	2506.86	7958.79	25267.6
3.08442	9.79243	31.089	98.7015	313.358	994.849	3158.45	10027.4	31835.2
								32768

811

1	3.1748	10.0794	32	101.594	322.54	1024	3251	10321.3
1.25992	4	12.6992	40.3175	128	406.375	1290.16	4096	13004
1.29684	4.11721	13.0713	41.4989	131.751	418.282	1327.96	4216.02	13385
1.33484	4.23785	13.4543	42.7149	135.611	430.539	1366.88	4339.56	13777.2
1.68179	5.33936	16.9514	53.8174	170.859	542.445	1722.16	5467.51	17358.2
1.73107	5.49581	17.4481	55.3943	175.866	558.34	1772.62	5627.71	17866.9
1.7818	5.65685	17.9594	57.0175	181.019	574.701	1824.56	5792.62	18390.4
2.24492	7.12719	22.6274	71.8376	228.07	724.077	2298.8	7298.24	23170.5
2.31071	7.33603	23.2904	73.9426	234.753	745.295	2366.16	7512.1	23849.4
2.37841	7.55099	23.9729	76.1093	241.632	767.133	2435.5	7732.22	24548.3
2.99661	9.51366	30.204	95.8917	304.437	966.528	3068.53	9741.98	30928.9
3.08442	9.79243	31.089	98.7015	313.358	994.849	3158.45	10027.4	31835.2
								32768

CHROMATIC "B " 226, 262, 622.

226

1	3.1748	0.0794	32	101.594	322.54	1024	3251	10321.3
1.05946	3.36359	10.6787	33.9028	107.635	341.719	1084.89	3444.31	10935
1.12246	3.5636	11.3137	35.9188	114.035	362.039	1149.4	3649.12	11585.2
1.33484	4.23785	13.4543	42.7149	135.611	430.539	1366.88	4339.56	13777.2
1.41421	4.48985	14.2544	45.2548	143.675	456.14	1448.15	4597.61	14596.5
1.49831	4.75683	15.102	47.9458	152.219	483.264	1534.27	4870.99	15464.4
1.7818	5.65685	17.9594	57.0175	181.019	574.701	1824.56	5792.62	18390.4
1.88775	5.99323	19.0273	60.4079	191.783	608.874	1933.06	6137.06	19484
2	6.3496	20.1587	64	203.187	645.079	2048	6502	20642.5
2.37841	7.55099	23.9729	76.1093	241.632	767.133	2435.5	7732.22	24548.3
2.51984	8	25.3984	80.635	256	812.75	2580.32	8192	26008
2.66968	8.4757	26.9087	85.4297	271.223	861.078	2733.75	8679.12	27554.5
								32768

2 6 2

1	3.1748	10.0794	32	101.594	322.54	1024	3251	10321.3
1.05946	3.36359	10.6787	33.9028	107.635	341.719	1084.89	3444.31	10935
1.25992	4	12.6992	40.3175	128	406.375	1290.16	4096	13004
1.33484	4.23785	13.4543	42.7149	135.611	430.539	1366.88	4339.56	13777.2
1.41421	4.48985	14.2544	45.2548	143.675	456.14	1448.15	4597.61	14596.5
1.68179	5.33936	16.9514	53.8174	170.859	542.445	1722.16	5467.51	17358.2
1.7818	5.65685	17.9594	57.0175	181.019	574.701	1824.56	5792.62	18390.4
1.88775	5.99323	19.0273	60.4079	191.783	608.874	1933.06	6137.06	19484
2.24492	7.12719	22.6274	71.8376	228.07	724.077	2298.8	7298.24	23170.5
2.37841	7.55099	23.9729	76.1093	241.632	767.133	2435.5	7732.22	24548.3
2.51984	8	25.3984	80.635	256	812.75	2580.32	8192	26008
2.99661	9.51366	30.204	95.8917	304.437	966.528	3068.53	9741.98	30928.9
								32768

6 2 2

1	3.1748	10.0794	32	101.594	322.54	1024	3251	10321.3
1.18921	3.7755	11.9865	38.0546	120.816	383.567	1217.75	3866.11	12274.1
1.25992	4	12.6992	40.3175	128	406.375	1290.16	4096	13004
1.33484	4.23785	13.4543	42.7149	135.611	430.539	1366.88	4339.56	13777.2
1.5874	5.03968	16	50.7968	161.27	512	1625.5	5160.64	16384
1.68179	5.33936	16.9514	53.8174	170.859	542.445	1722.16	5467.51	17358.2
1.7818	5.65685	17.9594	57.0175	181.019	574.701	1824.56	5792.62	18390.4
2.11893	6.72717	21.3574	67.8056	215.269	683.438	2169.78	6888.62	21870
2.24492	7.12719	22.6274	71.8376	228.07	724.077	2298.8	7298.24	23170.5
2.37841	7.55099	23.9729	76.1093	241.632	767.133	2435.5	7732.22	24548.3
2.82843	8.9797	28.5088	90.5097	287.35	912.28	2896.31	9195.21	29193
2.99661	9.51366	30.204	95.8917	304.437	966.528	3068.53	9741.98	30928.9
								32768

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“ Г ” - 334, 343, 433.

3 3 4

1	3.1748	10.0794	32	101.594	322.54	1024	3251	10321.3
1.09051	3.46215	10.9916	34.8962	110.789	351.732	1116.68	3545.24	11255.4
1.18921	3.7755	11.9865	38.0546	120.816	383.567	1217.75	3866.11	12274.1
1.33484	4.23785	13.4543	42.7149	135.611	430.539	1366.88	4339.56	13777.2
1.45565	4.62141	14.6721	46.5809	147.885	469.506	1490.59	4732.32	15024.2
1.5874	5.03968	16	50.7968	161.27	512	1625.5	5160.64	16384
1.7818	5.65685	17.9594	57.0175	181.019	574.701	1824.56	5792.62	18390.4
1.94306	6.16884	19.5849	62.1781	197.403	626.716	1989.7	6316.9	20054.9
2.11893	6.72717	21.3574	67.8056	215.269	683.438	2169.78	6888.62	21870
2.37841	7.55099	23.9729	76.1093	241.632	767.133	2435.5	7732.22	24548.3
2.59368	8.23442	26.1427	82.9977	263.501	836.565	2655.93	8432.05	26770.1
2.82843	8.9797	28.5088	90.5097	287.35	912.28	2896.31	9195.21	29193
								32768

3 4 3

1	3.1748	10.0794	32	01.594	322.54	1024	3251	10321.3
1.09051	3.46215	10.9916	34.8962	110.789	351.732	1116.68	3545.24	11255.4
1.22405	3.88613	12.3377	39.1697	124.356	394.806	1253.43	3979.39	12633.8
1.33484	4.23785	13.4543	42.7149	135.611	430.539	1366.88	4339.56	13777.2
1.45565	4.62141	14.6721	46.5809	147.885	469.506	1490.59	4732.32	15024.2
1.63392	5.18736	16.4688	52.2853	165.995	527.003	1673.13	5311.85	16864.1
1.7818	5.65685	17.9594	57.0175	181.019	574.701	1824.56	5792.62	18390.4
1.94306	6.16884	19.5849	62.1781	197.403	626.716	1989.7	6316.9	20054.9
2.18102	6.92429	21.9833	69.7925	221.577	703.464	2233.36	7090.48	22510.9
2.37841	7.55099	23.9729	76.1093	241.632	767.133	2435.5	7732.22	24548.3
2.59368	8.23442	26.1427	82.9977	263.501	836.565	2655.93	8432.05	26770.1
2.91131	9.24282	29.3441	93.1618	295.77	939.012	2981.18	9464.65	30048.4
								32768

4 3 3

1	3.1748	10.0794	32	101.594	322.54	1024	3251	10321.3
1.12246	3.5636	11.3137	35.9188	114.035	362.039	1149.4	3649.12	11585.2
1.22405	3.88613	12.3377	39.1697	124.356	394.806	1253.43	3979.39	12633.8
1.33484	4.23785	13.4543	42.7149	135.611	430.539	1366.88	4339.56	13777.2
1.49831	4.75683	15.102	47.9458	152.219	483.264	1534.27	4870.99	15464.4
1.63392	5.18736	16.4688	52.2853	165.995	527.003	1673.13	5311.85	16864.1
1.7818	5.65685	17.9594	57.0175	181.019	574.701	1824.56	5792.62	18390.4
2	6.3496	20.1587	64	203.187	645.079	2048	6502	20642.5
2.18102	6.92429	21.9833	69.7925	221.577	703.464	2233.36	7090.48	22510.9
2.37841	7.55099	23.9729	76.1093	241.632	767.133	2435.5	7732.22	24548.3
2.66968	8.4757	26.9087	85.4297	271.223	861.078	2733.75	8679.12	27554.5
2.91131	9.24282	29.3441	93.1618	295.77	939.012	2981.18	9464.65	30048.4
								32768

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DIATONIC “△” 442, 424, 244.

4 4 2

1	3.1748	10.0794	32	101.594	322.54	1024	3251	10321.3
1.12246	3.5636	11.3137	35.9188	114.035	362.039	1149.4	3649.12	11585.2
1.25992	4	12.6992	40.3175	128	406.375	1290.16	4096	13004
1.33484	4.23785	13.4543	42.7149	135.611	430.539	1366.88	4339.56	13777.2
1.49831	4.75683	15.102	47.9458	152.219	483.264	1534.27	4870.99	15464.4
1.68179	5.33936	16.9514	53.8174	170.859	542.445	1722.16	5467.51	17358.2
1.7818	5.65685	17.9594	57.0175	181.019	574.701	1824.56	5792.62	18390.4
2	6.3496	20.1587	64	203.187	645.079	2048	6502	20642.5
2.24492	7.12719	22.6274	71.8376	228.07	724.077	2298.8	7298.24	23170.5
2.37841	7.55099	23.9729	76.1093	241.632	767.133	2435.5	7732.22	24548.3
2.66968	8.4757	26.9087	85.4297	271.223	861.078	2733.75	8679.12	27554.5
2.99661	9.51366	30.204	95.8917	304.437	966.528	3068.53	9741.98	30928.9
								32768

4 2 4

1	3.1748	10.0794	32	101.594	322.54	1024	3251	10321.3
1.12246	3.5636	11.3137	35.9188	114.035	362.039	1149.4	3649.12	11585.2
1.18921	3.7755	11.9865	38.0546	120.816	383.567	1217.75	3866.11	12274.1
1.33484	4.23785	13.4543	42.7149	135.611	430.539	1366.88	4339.56	13777.2
1.49831	4.75683	15.102	47.9458	152.219	483.264	1534.27	4870.99	15464.4
1.5874	5.03968	16	50.7968	161.27	512	1625.5	5160.64	16384
1.7818	5.65685	17.9594	57.0175	181.019	574.701	1824.56	5792.62	18390.4
2	6.3496	20.1587	64	203.187	645.079	2048	6502	20642.5
2.11893	6.72717	21.3574	67.8056	215.269	683.438	2169.78	6888.62	21870
2.37841	7.55099	23.9729	76.1093	241.632	767.133	2435.5	7732.22	24548.3
2.66968	8.4757	26.9087	85.4297	271.223	861.078	2733.75	8679.12	27554.5
2.82843	8.9797	28.5088	90.5097	287.35	912.28	2896.31	9195.21	29193
								32768

2 4 4

1	3.1748	10.0794	32	101.594	322.54	1024	3251	10321.3
1.05946	3.36359	10.6787	33.9028	107.635	341.719	1084.89	3444.31	10935
1.18921	3.7755	11.9865	38.0546	120.816	383.567	1217.75	3866.11	12274.1
1.33484	4.23785	13.4543	42.7149	135.611	430.539	1366.88	4339.56	13777.2
1.41421	4.48985	14.2544	45.2548	143.675	456.14	1448.15	4597.61	14596.5
1.5874	5.03968	16	50.7968	161.27	512	1625.5	5160.64	16384
1.7818	5.65685	17.9594	57.0175	181.019	574.701	1824.56	5792.62	18390.4
1.88775	5.99323	19.0273	60.4079	191.783	608.874	1933.06	6137.06	19484
2.11893	6.72717	21.3574	67.8056	215.269	683.438	2169.78	6888.62	21870
2.37841	7.55099	23.9729	76.1093	241.632	767.133	2435.5	7732.22	24548.3
2.51984	8	25.3984	80.635	256	812.75	2580.32	8192	26008
2.82843	8.9797	28.5088	90.5097	287.35	912.28	2896.31	9195.21	29193
								32768

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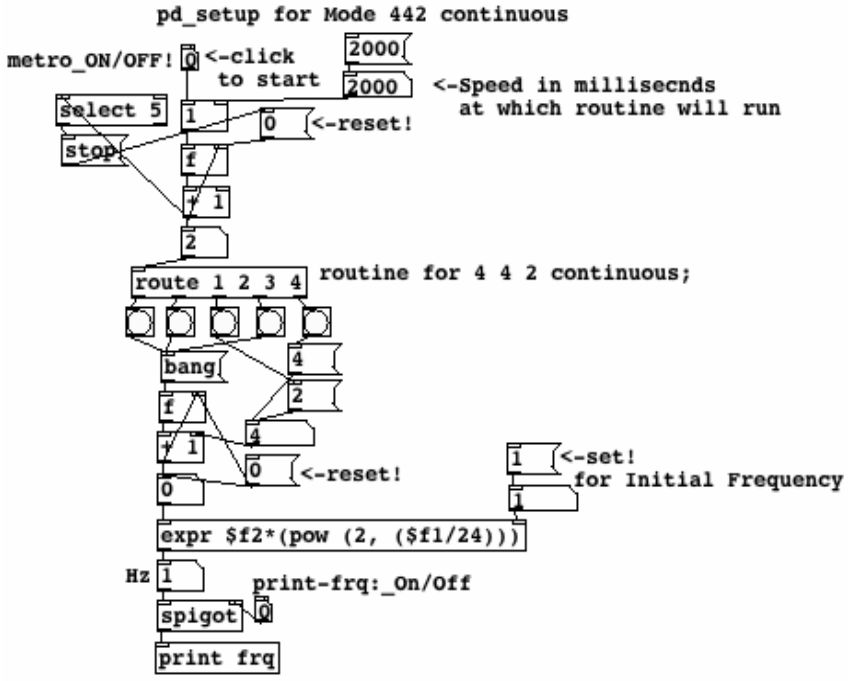
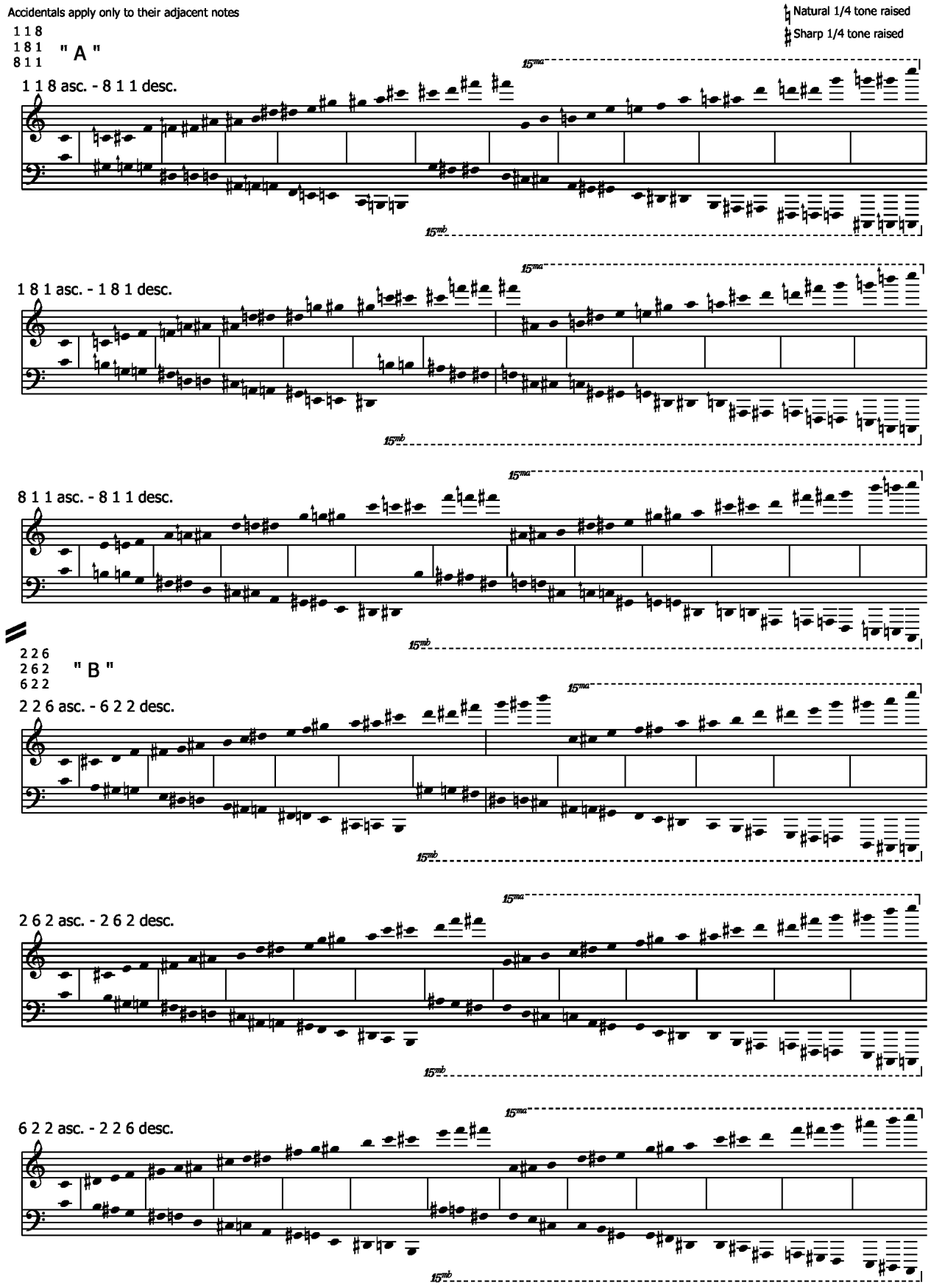
The graphic below shows a pure data (pd) routine to calculate the Hertz values for "Δ" mode – 442, repeated continuously.

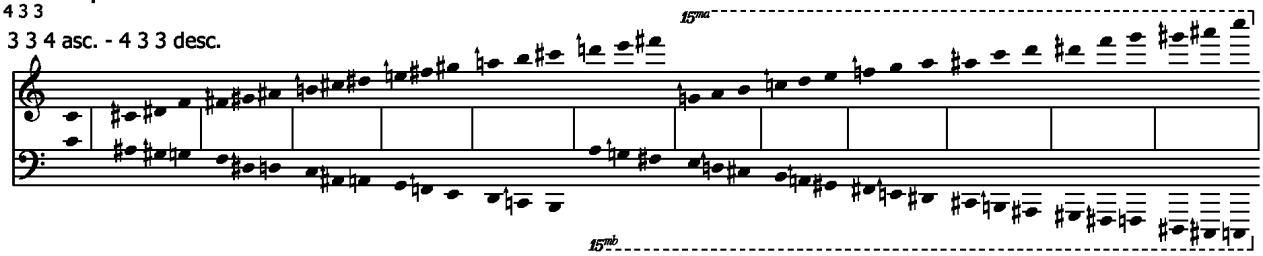


The scales below use the same continuous interval steps of the ancient modes and their permutations, written in tempered notation based on the central note c´ – MIDI no. C4, ascending and descending. One central note example out of the 24 quarter note “tonics” available in the “tempered” octave.

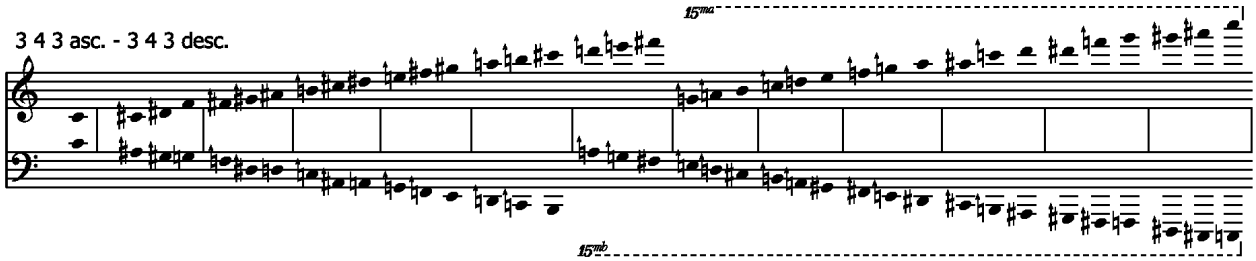

Accidentals apply only to their adjacent notes
Natural 1/4 tone raised
Sharp 1/4 tone raised
1 1 8
1 8 1
8 1 1
" A "
1 1 8 asc. - 8 1 1 desc.
1 8 1 asc. - 1 8 1 desc.
8 1 1 asc. - 8 1 1 desc.
2 2 6
2 6 2
6 2 2
" B "
2 2 6 asc. - 6 2 2 desc.
2 6 2 asc. - 2 6 2 desc.
6 2 2 asc. - 2 2 6 desc.

3 3 4
3 4 3 " Γ "
4 3 3

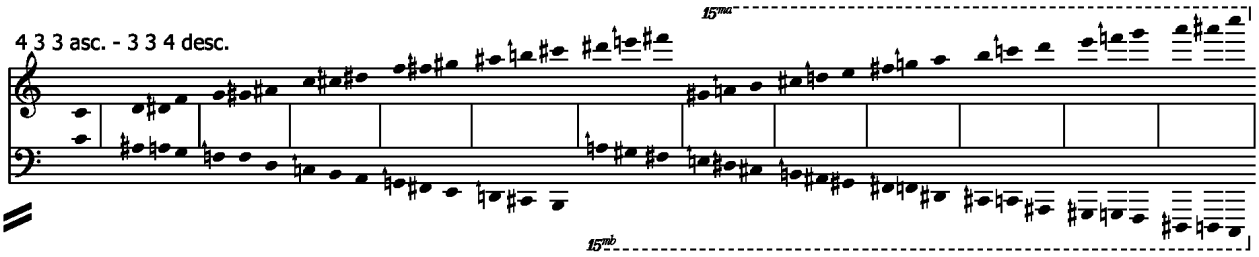
3 3 4 asc. - 4 3 3 desc.



3 4 3 asc. - 3 4 3 desc.

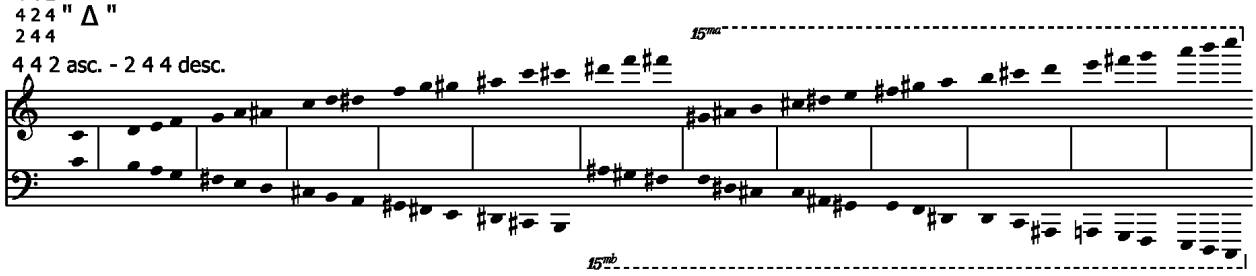


4 3 3 asc. - 3 3 4 desc.

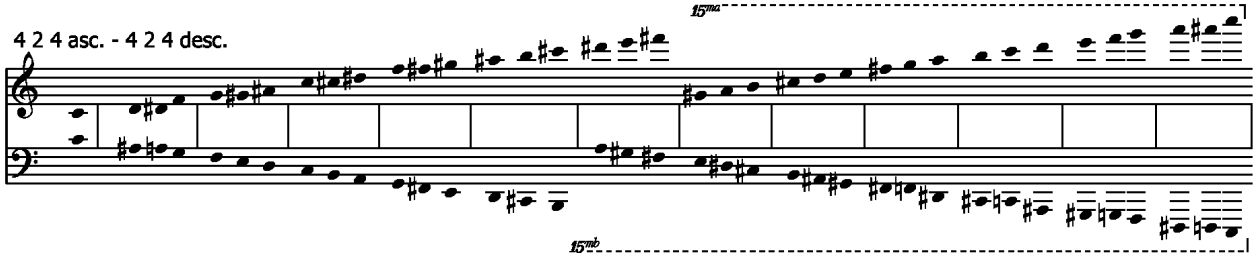


4 4 2
4 2 4 " Δ "
2 4 4

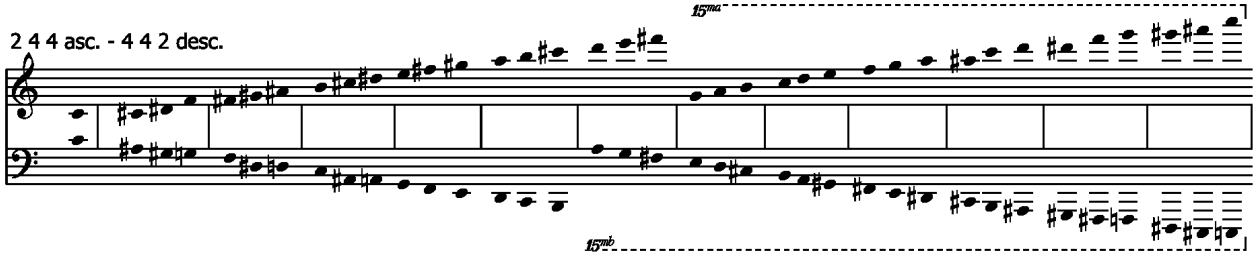
4 4 2 asc. - 2 4 4 desc.



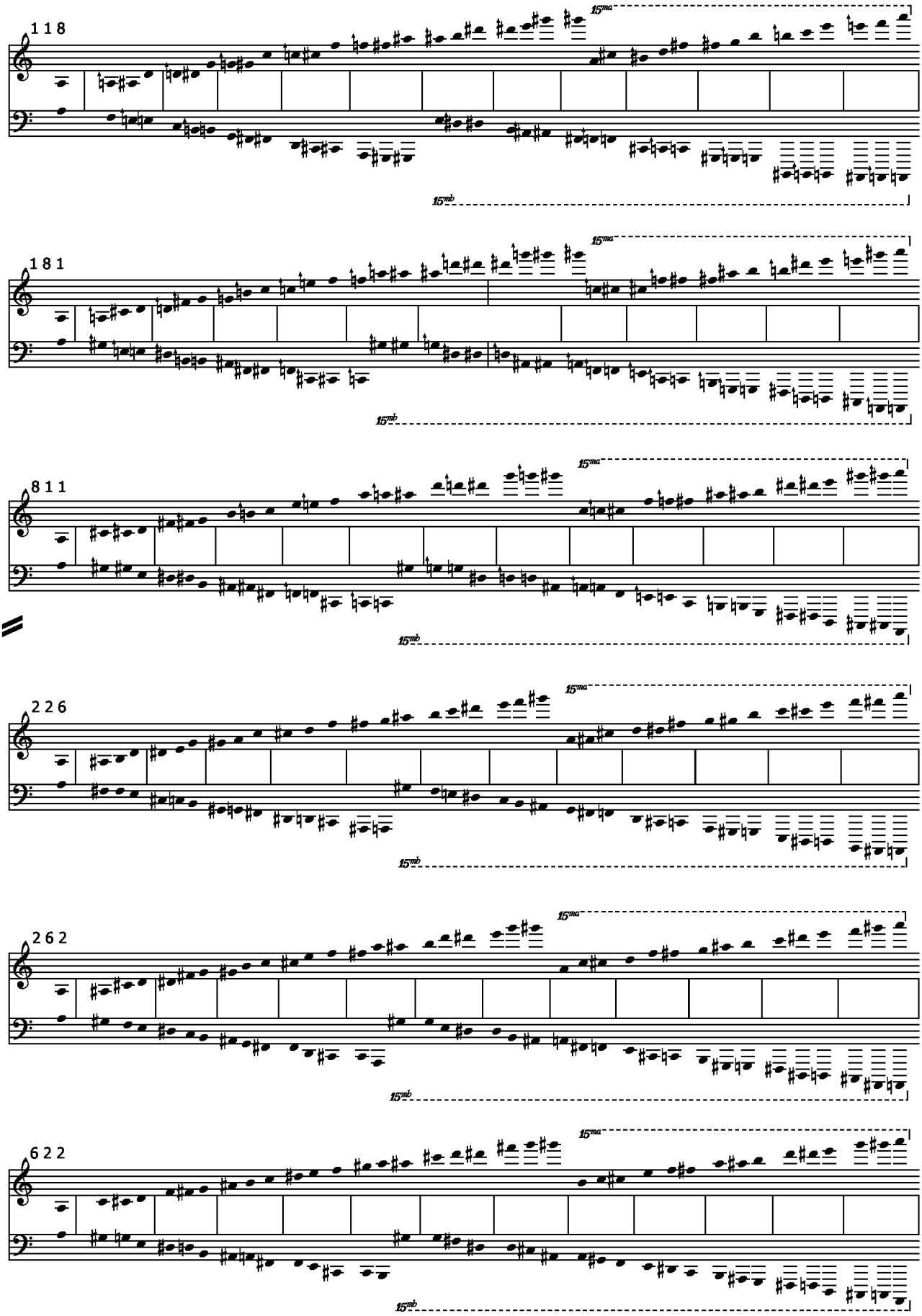
4 2 4 asc. - 4 2 4 desc.



2 4 4 asc. - 4 4 2 desc.



Below the same modes on central note a - MIDI A3



3 3 4
15ma
15mb
3 4 3
15ma
15mb
4 3 3
15ma
15mb
4 4 2
15ma
15mb
4 2 4
15ma
15mb
2 4 4
15ma
15mb

In the above scales, all 24 notes of the ¼ tone octave are accounted for but distributed across the complete set of twelve modes. Depending on the mode itself (118 or 334 etc.) not all 24 notes are present.
Modes 226, 262, 622, 442, 242 & 244 have no quarter tones. The scales/modes can also be regarded as pitch class sets for the notes available within whatever particular chosen mode. There are 24 tones/tonics available upon which to construct a "universe" of scales based upon these tetrachordal interval modes.
All non quarter-tone notes have their equivalent MIDI no. and many MIDI protocols also allow for quarter tone pitch changes within the tempered system.

Below is a simple table to assist counting intervals. The ^ symbol indicates a quarter-tone raising i.e. c^ = c natural ¼ tone higher: c#^ = c sharp ¼ tone higher etc.
The column nos. at the top of the table beside the "note" column indicate the ¼ tone intervals 1, 2, 3, 4, 6 & 8, as in the ancient modes.

NOTE	1	2	3	4	6	8
c	c^	c#	c#^	d	d#	e
c^	c#	c#^	d	d^	d#^	e^
c#	c#^	d	d^	d#	e	f
c#^	d	d^	d#	d#^	e^	f^
d	d^	d#	d#^	e	f	f#
d^	d#	d#^	e	e^	f^	f#^
d#	d#^	e	e^	f	f#	g
d#^	e	e^	f	f^	f#^	g^
e	e^	f	f^	f#	g	g#
e^	f	f^	f#	f#^	g^	g#^
f	f^	f#	f#^	g	g#	a
f^	f#	f#^	g	g^	g#^	a^
f#	f#^	g	g^	g#	a	a#
f#^	g	g^	g#	g#^	a^	a#^
g	g^	g#	g#^	a	a#	b
g^	g#	g#^	a	a^	a#^	b^
g#	g#^	a	a^	a#	b	c
g#^	a	a^	a#	a#^	b^	c^
a	a^	a#	a#^	b	c	c#
a^	a#	a#^	b	b^	c^	c#^
a#	a#^	b	b^	c	c#	d
a#^	b	b^	c	c^	c#^	d^
b	b^	c	c^	c#	d	d#
b^	c	c^	c#	c#^	d^	d#^

For electronic/electroacoustic works, any frequency can be chosen as a starting point upon which to construct such modal scales using the increments of the intervals as expressed by the ancient modes (811, 442 etc.), or frequencies from the Natural Interval Series, or frequencies as shown in the tables above starting from 1- 1048576Hz can be utilized.

Below are frequency lists for all modes 118 etc. as per harmonic series expressed mathematically; expr $f2*(pow (2, ($f1/24))) arranged as octave "scales" adding the extra interval of 4 to reach the octave and arranged/permutated as the intervals as in the table shown to the right, the same table as used to create the "baroque-church-modes" for the modes above.

Enharmonic A	Chromatic B	Γ	Diatonic Δ
118	226	334	442
118118-4	226226-4	334334-4	442442-4
181181-4	262262-4	343343-4	424424-4
811811-4	622622-4	433433-4	244244-4

The increase in frequency is read horizontally and each new line starts at the next octave.
The fundamental frequency is 1 and all have been calculated for an equal tempered 24 step/tone octave.

----1 1 8 1 1 8 4----

1	1.0293	1.05946	1.33484	1.37395	1.41421	1.7818
2	2.0586	2.11893	2.66968	2.74791	2.82843	3.5636
4	4.11721	4.23785	5.33936	5.49581	5.65685	7.12719
8	8.23442	8.4757	10.6787	10.9916	11.3137	14.2544
16	16.4688	16.9514	21.3574	21.9833	22.6274	28.5088
32	32.9377	33.9028	42.7149	43.9665	45.2548	57.0175
64	65.8753	67.8056	85.4297	87.933	90.5097	114.035
128	131.751	135.611	170.859	175.866	181.019	228.07
256	263.501	271.223	341.719	351.732	362.039	456.14
512	527.003	542.445	683.438	703.464	724.077	912.28
1024	1054.01	1084.89	1366.88	1406.93	1448.15	1824.56
2048	2108.01	2169.78	2733.75	2813.86	2896.31	3649.12
4096	4216.02	4339.56	5467.51	5627.71	5792.62	7298.24
8192	8432.05	8679.12	10935	11255.4	11585.2	14596.5
16384	16864.1	17358.2	21870	22510.9	23170.5	29193
32768	33728.2	34716.5	43740	45021.7	46340.9	58385.9
65536	67456.3	69433	87480	90043.5	92681.9	116772
131072	134913	138866	174960	180087	185364	233544
262144	269825	277732	349920	360174	370728	467088
524288	539651	555464	699840	720348	741455	934175

----2 2 6 2 2 6 4----

1	1.05946	1.12246	1.33484	1.41421	1.49831	1.7818
2	2.11893	2.24492	2.66968	2.82843	2.99661	3.5636
4	4.23785	4.48985	5.33936	5.65685	5.99323	7.12719
8	8.4757	8.9797	10.6787	11.3137	11.9865	14.2544
16	16.9514	17.9594	21.3574	22.6274	23.9729	28.5088
32	33.9028	35.9188	42.7149	45.2548	47.9458	57.0175
64	67.8056	71.8376	85.4297	90.5097	95.8917	114.035
128	135.611	143.675	170.859	181.019	191.783	228.07
256	271.223	287.35	341.719	362.039	383.567	456.14
512	542.445	574.701	683.438	724.077	767.133	912.28
1024	1084.89	1149.4	1366.88	1448.15	1534.27	1824.56
2048	2169.78	2298.8	2733.75	2896.31	3068.53	3649.12
4096	4339.56	4597.61	5467.51	5792.62	6137.06	7298.24
8192	8679.12	9195.21	10935	11585.2	12274.1	14596.5
16384	17358.2	18390.4	21870	23170.5	24548.3	29193
32768	34716.5	36780.8	43740	46340.9	49096.5	58385.9
65536	69433	73561.6	87480	92681.9	98193.1	116772
131072	138866	147123	174960	185364	196386	233544
262144	277732	294247	349920	370728	392772	467088
524288	555464	588493	699840	741455	785545	934175

----1 8 1 1 8 1 4----

1	1.0293	1.29684	1.33484	1.37395	1.73107	1.7818
2	2.0586	2.59368	2.66968	2.74791	3.46215	3.5636
4	4.11721	5.18736	5.33936	5.49581	6.92429	7.12719
8	8.23442	10.3747	10.6787	10.9916	13.8486	14.2544
16	16.4688	20.7494	21.3574	21.9833	27.6972	28.5088
32	32.9377	41.4989	42.7149	43.9665	55.3943	57.0175
64	65.8753	82.9977	85.4297	87.933	110.789	114.035
128	131.751	165.995	170.859	175.866	221.577	228.07
256	263.501	331.991	341.719	351.732	443.155	456.14
512	527.003	663.982	683.438	703.464	886.31	912.28
1024	1054.01	1327.96	1366.88	1406.93	1772.62	1824.56
2048	2108.01	2655.93	2733.75	2813.86	3545.24	3649.12
4096	4216.02	5311.85	5467.51	5627.71	7090.48	7298.24
8192	8432.05	10623.7	10935	11255.4	14181	14596.5
16384	16864.1	21247.4	21870	22510.9	28361.9	29193
32768	33728.2	42494.8	43740	45021.7	56723.8	58385.9
65536	67456.3	84989.7	87480	90043.5	113448	116772
131072	134913	169979	174960	180087	226895	233544
262144	269825	339959	349920	360174	453790	467088
524288	539651	679917	699840	720348	907580	934175

----2 6 2 2 6 2 4----

1	1.05946	1.25992	1.33484	1.41421	1.68179	1.7818
2	2.11893	2.51984	2.66968	2.82843	3.36359	3.5636
4	4.23785	5.03968	5.33936	5.65685	6.72717	7.12719
8	8.4757	10.0794	10.6787	11.3137	13.4543	14.2544
16	16.9514	20.1587	21.3574	22.6274	26.9087	28.5088
32	33.9028	40.3175	42.7149	45.2548	53.8174	57.0175
64	67.8056	80.635	85.4297	90.5097	107.635	114.035
128	135.611	161.27	170.859	181.019	215.269	228.07
256	271.223	322.54	341.719	362.039	430.539	456.14
512	542.445	645.079	683.438	724.077	861.078	912.28
1024	1084.89	1290.16	1366.88	1448.15	1722.16	1824.56
2048	2169.78	2580.32	2733.75	2896.31	3444.31	3649.12
4096	4339.56	5160.64	5467.51	5792.62	6888.62	7298.24
8192	8679.12	10321.3	10935	11585.2	13777.2	14596.5
16384	17358.2	20642.5	21870	23170.5	27554.5	29193
32768	34716.5	41285.1	43740	46340.9	55109	58385.9
65536	69433	82570.2	87480	92681.9	110218	116772
131072	138866	165140	174960	185364	220436	233544
262144	277732	330281	349920	370728	440872	467088
524288	555464	660562	699840	741455	881744	934175

----8 1 1 8 1 1 4----

1	1.25992	1.29684	1.33484	1.68179	1.73107	1.7818
2	2.51984	2.59368	2.66968	3.36359	3.46215	3.5636
4	5.03968	5.18736	5.33936	6.72717	6.92429	7.12719
8	10.0794	10.3747	10.6787	13.4543	13.8486	14.2544
16	20.1587	20.7494	21.3574	26.9087	27.6972	28.5088
32	40.3175	41.4989	42.7149	53.8174	55.3943	57.0175
64	80.635	82.9977	85.4297	107.635	110.789	114.035
128	161.27	165.995	170.859	215.269	221.577	228.07
256	322.54	331.991	341.719	430.539	443.155	456.14
512	645.079	663.982	683.438	861.078	886.31	912.28
1024	1290.16	1327.96	1366.88	1722.16	1772.62	1824.56
2048	2580.32	2655.93	2733.75	3444.31	3545.24	3649.12
4096	5160.64	5311.85	5467.51	6888.62	7090.48	7298.24
8192	10321.3	10623.7	10935	13777.2	14181	14596.5
16384	20642.5	21247.4	21870	27554.5	28361.9	29193
32768	41285.1	42494.8	43740	55109	56723.8	58385.9
65536	82570.2	84989.7	87480	110218	113448	116772
131072	165140	169979	174960	220436	226895	233544
262144	330281	339959	349920	440872	453790	467088
524288	660562	679917	699840	881744	907580	934175

----6 2 2 6 2 2 4----

1	1.18921	1.25992	1.33484	1.5874	1.68179	1.7818
2	2.37841	2.51984	2.66968	3.1748	3.36359	3.5636
4	4.75683	5.03968	5.33936	6.3496	6.72717	7.12719
8	9.51366	10.0794	10.6787	12.6992	13.4543	14.2544
16	19.0273	20.1587	21.3574	25.3984	26.9087	28.5088
32	38.0546	40.3175	42.7149	50.7968	53.8174	57.0175
64	76.1093	80.635	85.4297	101.594	107.635	114.035
128	152.219	161.27	170.859	203.187	215.269	228.07
256	304.437	322.54	341.719	406.375	430.539	456.14
512	608.874	645.079	683.438	812.75	861.078	912.28
1024	1217.75	1290.16	1366.88	1625.5	1722.16	1824.56
2048	2435.5	2580.32	2733.75	3251	3444.31	3649.12
4096	4870.99	5160.64	5467.51	6502	6888.62	7298.24
8192	9741.98	10321.3	10935	13004	13777.2	14596.5
16384	19484	20642.5	21870	26008	27554.5	29193
32768	38967.9	41285.1	43740	52016	55109	58385.9
65536	77935.9	82570.2	87480	104032	110218	116772
131072	155872	165140	174960	208064	220436	233544
262144	311744	330281	349920	416127	440872	467088
524288	623487	660562	699840	832255	881744	934175

----3 3 4 3 3 4 4----

1	1.09051	1.18921	1.33484	1.45565	1.5874	1.7818
2	2.18102	2.37841	2.66968	2.91131	3.1748	3.5636
4	4.36203	4.75683	5.33936	5.82261	6.3496	7.12719
8	8.72406	9.51366	10.6787	11.6452	12.6992	14.2544
16	17.4481	19.0273	21.3574	23.2904	25.3984	28.5088
32	34.8962	38.0546	42.7149	46.5809	50.7968	57.0175
64	69.7925	76.1093	85.4297	93.1618	101.594	114.035
128	139.585	152.219	170.859	186.324	203.187	228.07
256	279.17	304.437	341.719	372.647	406.375	456.14
512	558.34	608.874	683.438	745.295	812.75	912.28
1024	1116.68	1217.75	1366.88	1490.59	1625.5	1824.56
2048	2233.36	2435.5	2733.75	2981.18	3251	3649.12
4096	4466.72	4870.99	5467.51	5962.36	6502	7298.24
8192	8933.44	9741.98	10935	11924.7	13004	14596.5
16384	17866.9	19484	21870	23849.4	26008	29193
32768	35733.8	38967.9	43740	47698.9	52016	58385.9
65536	71467.5	77935.9	87480	95397.6	104032	116772
131072	142935	155872	174960	190795	208064	233544
262144	285870	311744	349920	381591	416127	467088
524288	571740	623487	699840	763181	832255	934175

----4 4 2 4 4 2 4----

1	1.12246	1.25992	1.33484	1.49831	1.68179	1.7818
2	2.24492	2.51984	2.66968	2.99661	3.36359	3.5636
4	4.48985	5.03968	5.33936	5.99323	6.72717	7.12719
8	8.9797	10.0794	10.6787	11.9865	13.4543	14.2544
16	17.9594	20.1587	21.3574	23.9729	26.9087	28.5088
32	35.9188	40.3175	42.7149	47.9458	53.8174	57.0175
64	71.8376	80.635	85.4297	95.8917	107.635	114.035
128	143.675	161.27	170.859	191.783	215.269	228.07
256	287.35	322.54	341.719	383.567	430.539	456.14
512	574.701	645.079	683.438	767.133	861.078	912.28
1024	1149.4	1290.16	1366.88	1534.27	1722.16	1824.56
2048	2298.8	2580.32	2733.75	3068.53	3444.31	3649.12
4096	4597.61	5160.64	5467.51	6137.06	6888.62	7298.24
8192	9195.21	10321.3	10935	12274.1	13777.2	14596.5
16384	18390.4	20642.5	21870	24548.3	27554.5	29193
32768	36780.8	41285.1	43740	49096.5	55109	58385.9
65536	73561.6	82570.2	87480	98193.1	110218	116772
131072	147123	165140	174960	196386	220436	233544
262144	294247	330281	349920	392772	440872	467088
524288	588493	660562	699840	785545	881744	934175

----------3 4 3 3 4 3 4---------

1	1.09051	1.22405	1.33484	1.45565	1.63392	1.7818
2	2.18102	2.44811	2.66968	2.91131	3.26783	3.5636
4	4.36203	4.89621	5.33936	5.82261	6.53566	7.12719
8	8.72406	9.79243	10.6787	11.6452	13.0713	14.2544
16	17.4481	19.5849	21.3574	23.2904	26.1427	28.5088
32	34.8962	39.1697	42.7149	46.5809	52.2853	57.0175
64	69.7925	78.3394	85.4297	93.1618	104.571	114.035
128	139.585	156.679	170.859	186.324	209.141	228.07
256	279.17	313.358	341.719	372.647	418.282	456.14
512	558.34	626.716	683.438	745.295	836.565	912.28
1024	1116.68	1253.43	1366.88	1490.59	1673.13	1824.56
2048	2233.36	2506.86	2733.75	2981.18	3346.26	3649.12
4096	4466.72	5013.72	5467.51	5962.36	6692.52	7298.24
8192	8933.44	10027.4	10935	11924.7	13385	14596.5
16384	17866.9	20054.9	21870	23849.4	26770.1	29193
32768	35733.8	40109.8	43740	47698.9	53540.1	58385.9
65536	71467.5	80219.5	87480	95397.6	107080	116772
131072	142935	160439	174960	190795	214161	233544
262144	285870	320878	349920	381591	428321	467088
524288	571740	641756	699840	763181	856643	934175

----4 2 4 4 2 4 4----

1	1.12246	1.18921	1.33484	1.49831	1.5874	1.7818
2	2.24492	2.37841	2.66968	2.99661	3.1748	3.5636
4	4.48985	4.75683	5.33936	5.99323	6.3496	7.12719
8	8.9797	9.51366	10.6787	11.9865	12.6992	14.2544
16	17.9594	19.0273	21.3574	23.9729	25.3984	28.5088
32	35.9188	38.0546	42.7149	47.9458	50.7968	57.0175
64	71.8376	76.1093	85.4297	95.8917	101.594	114.035
128	143.675	152.219	170.859	191.783	203.187	228.07
256	287.35	304.437	341.719	383.567	406.375	456.14
512	574.701	608.874	683.438	767.133	812.75	912.28
1024	1149.4	1217.75	1366.88	1534.27	1625.5	1824.56
2048	2298.8	2435.5	2733.75	3068.53	3251	3649.12
4096	4597.61	4870.99	5467.51	6137.06	6502	7298.24
8192	9195.21	9741.98	10935	12274.1	13004	14596.5
16384	18390.4	19484	21870	24548.3	26008	29193
32768	36780.8	38967.9	43740	49096.5	52016	58385.9
65536	73561.6	77935.9	87480	98193.1	104032	116772
131072	147123	155872	174960	196386	208064	233544
262144	294247	311744	349920	392772	416127	467088
524288	588493	623487	699840	785545	832255	934175

----4 3 3 4 3 3 4----

1	1.12246	1.22405	1.33484	1.49831	1.63392	1.7818
2	2.24492	2.44811	2.66968	2.99661	3.26783	3.5636
4	4.48985	4.89621	5.33936	5.99323	6.53566	7.12719
8	8.9797	9.79243	10.6787	11.9865	13.0713	14.2544
16	17.9594	19.5849	21.3574	23.9729	26.1427	28.5088
32	35.9188	39.1697	42.7149	47.9458	52.2853	57.0175
64	71.8376	78.3394	85.4297	95.8917	104.571	114.035
128	143.675	156.679	170.859	191.783	209.141	228.07
256	287.35	313.358	341.719	383.567	418.282	456.14
512	574.701	626.716	683.438	767.133	836.565	912.28
1024	1149.4	1253.43	1366.88	1534.27	1673.13	1824.56
2048	2298.8	2506.86	2733.75	3068.53	3346.26	3649.12
4096	4597.61	5013.72	5467.51	6137.06	6692.52	7298.24
8192	9195.21	10027.4	10935	12274.1	13385	14596.5
16384	18390.4	20054.9	21870	24548.3	26770.1	29193
32768	36780.8	40109.8	43740	49096.5	53540.1	58385.9
65536	73561.6	80219.5	87480	98193.1	107080	116772
131072	147123	160439	174960	196386	214161	233544
262144	294247	320878	349920	392772	428321	467088
524288	588493	641756	699840	785545	856643	934175

----2 4 4 2 4 4 4----

1	1.05946	1.18921	1.33484	1.41421	1.5874	1.7818
2	2.11893	2.37841	2.66968	2.82843	3.1748	3.5636
4	4.23785	4.75683	5.33936	5.65685	6.3496	7.12719
8	8.4757	9.51366	10.6787	11.3137	12.6992	14.2544
16	16.9514	19.0273	21.3574	22.6274	25.3984	28.5088
32	33.9028	38.0546	42.7149	45.2548	50.7968	57.0175
64	67.8056	76.1093	85.4297	90.5097	101.594	114.035
128	135.611	152.219	170.859	181.019	203.187	228.07
256	271.223	304.437	341.719	362.039	406.375	456.14
512	542.445	608.874	683.438	724.077	812.75	912.28
1024	1084.89	1217.75	1366.88	1448.15	1625.5	1824.56
2048	2169.78	2435.5	2733.75	2896.31	3251	3649.12
4096	4339.56	4870.99	5467.51	5792.62	6502	7298.24
8192	8679.12	9741.98	10935	11585.2	13004	14596.5
16384	17358.2	19484	21870	23170.5	26008	29193
32768	34716.5	38967.9	43740	46340.9	52016	58385.9
65536	69433	77935.9	87480	92681.9	104032	116772
131072	138866	155872	174960	185364	208064	233544
262144	277732	311744	349920	370728	416127	467088
524288	555464	623487	699840	741455	832255	934175

Modes/Systems/Species ascending & descending scales - ΔPOMOI - (Road-Ways)

" **A** " ("Enharmonic")

1 1 8 1 1 8 - 4

1 8 1 1 8 1 - 4

8 1 1 8 1 1 - 4

" **B** " ("Chromatic")

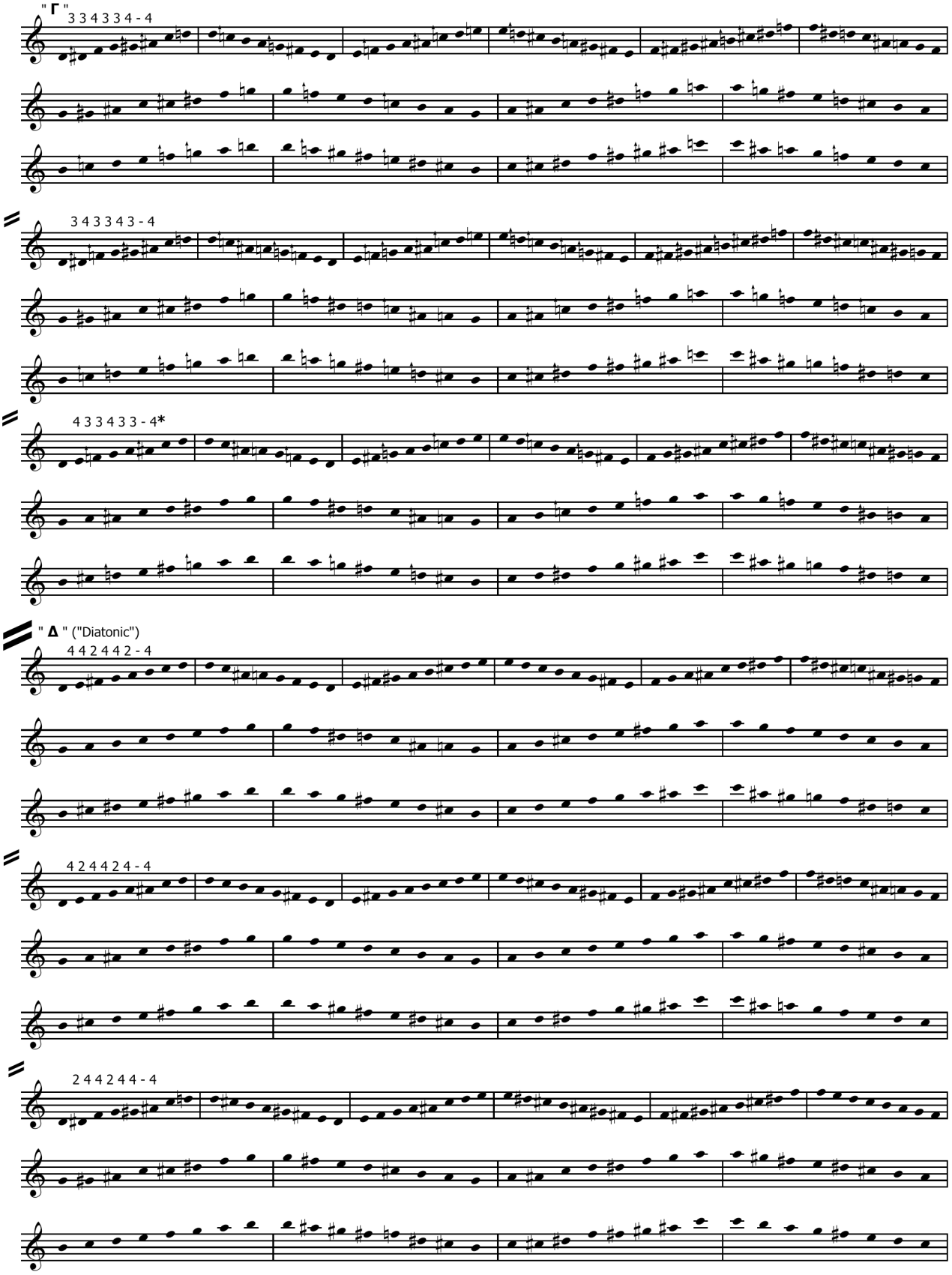
2 2 6 2 2 6 - 4

2 6 2 2 6 2 - 4

6 2 2 6 2 2 - 4

1 1 8 1 1 8 1 1 8 1 1 8 4
1 8 1 1 8 1 1 8 1 1 8 1 4
8 1 1 8 1 1 8 1 1 8 1 1 4

2 2 6 2 2 6 2 2 6 2 2 6 4
2 6 2 2 6 2 2 6 2 2 6 2 4
6 2 2 6 2 2 6 2 2 6 2 2 4



3 3 4 3 3 4 3 4 4
3 4 3 3 4 3 3 4
4 3 3 3 4 3 3 4

*

4 3 3 4 3 3 - 4 mode
results with exactly the same notes,
in ascending and descending order.

4 4 2 4 4 2 4 4 2 4
2 4 2 2 4 2 4 2
4 2 2 4 2 2 4 2

"...Moreover, so much of music as is adapted
to the sound of the voice and to the sense of hearing
is granted to us for the sake of harmony;
and harmony, which has motions
akin to the revolutions of our souls,
is not regarded by the intelligent votary of the Muses
as given by them with a view to irrational pleasure,
which is deemed to be the purpose of it in our day,
but as meant to correct any discord which may have arisen
in the courses of the soul,
and to be our ally
in bringing her into harmony
and agreement with herself;
and rhythm too was given by them
for the same reason,
on account of the irregular and graceless ways
which prevail among mankind generally,
and to help us against them. ..."

Plato "Timaeus", Rev. Jowett´s translation

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"It is the business of the musician
to harmonize every instrument..."

Demophilios

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